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Exploring Grade XI Students' Pattern Recognition in Solving Complex Number Exponentiation Problems: A Qualitative Study

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ABSTRACT

Purpose – This study aims to explain students' ability to recognize patterns (pattern recognition) when solving complex number exponentiation problems, especially in shapes $(1 + i)^n$ with great rank. The focus of the research is directed at how students identify cyclic patterns and use those structures to simplify calculations. **Methods –** This study employed a descriptive qualitative approach involving three Grade XI students selected through purposive sampling. Data were collected through problem-solving tests, students' written work documentation, controlled think-aloud protocols, and semi-structured interviews. The data were analyzed using interactive qualitative analysis consisting of data reduction, data display, and conclusion drawing to identify students' pattern recognition processes in solving complex number exponentiation problems. **Findings –** The findings revealed that all participants successfully recognized the cyclic power pattern of the imaginary unit i, namely $i, -1, -i$, and 1, which repeats every four powers. Students used this cyclic structure together with modulo 4 reasoning to simplify high-order exponentiation problems, such as determining the value of i^{1013} and simplifying expressions involving $(1 + i)^{2026}$. Although the participants employed different solution strategies, all were able to identify the same underlying mathematical structure and arrive at correct solutions. These results indicate that pattern recognition plays a central role in supporting students' mathematical reasoning and efficient problem-solving in complex number exponentiation. **Research implications –** These findings indicate that learning mathematics in complex number materials needs to emphasize the development of pattern recognition skills so that students can more easily understand abstract concepts and reduce computational burden. **Originality –** This research makes a new contribution by highlighting specifically how complex number cyclic patterns are used by students to solve large ranks, as well as mapping the variation of strategies that arise in the mathematical reasoning process.

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INTRODUCTION

The concept of complex numbers is fundamental in modern mathematics and plays an important role in analysis, algebra, and engineering applications. Complex numbers are defined as numbers that consist of real and imaginary parts as described by Spatschek (2026) in his work on exponential functions. In mathematics, the exponential of complex numbers has become an increasingly broad topic because it is related to rotational patterns in complex planes. These patterns are important for helping students understand how complex numbers behave in certain power operations. According to Heuvel-Panhuizen & Drijvers (2020) understanding abstract concepts such as complex numbers requires the presence of pattern recognition to build a strong mental representation. The exponential pattern of complex numbers often appears periodically so that they can be analyzed through relations between ranks. Thus, students' ability to recognize patterns is an important aspect in solving complex number exponentiation problems. This topic is becoming increasingly relevant because exponentiation patterns are the foundation for understanding geometric transformations in complex planes. This study emphasizes research gaps that are relevant to the topic of exponential complex numbers. Today, some sections tend to repeat the statement that there has been no research examining pattern recognition in the exponential of complex numbers from the perspective of students' thought processes, but such claims have not been supported by an adequate literature review. In addition, some of the references used are less directly related to the context of the study because they discuss pattern recognition in general, data visualization, or computation, rather than students' mathematical reasoning on complex numbers. Therefore, the literature review must focus on mathematics education research that specifically discusses students' understanding of complex numbers, imaginary unit power cycle patterns, modular *reasoning*, pattern generalization, and conceptual difficulties.

The concept of complex numbers has been taught at the school and college levels, many students still have difficulty in understanding the exponential process. The difficulty arises due to the nature of complex numbers that are not intuitive compared to real numbers as stated by (Gvozdic & Sander, 2018). Students often fail to see repeating patterns in complex number powers due to a lack of representation and visualization skills. They also tend to use mechanical procedures without understanding the mathematical structure behind the exponential process. According to Yasin & Nusantara (2023) The inability to recognize patterns is one of the main sources of failure in mathematical problem-solving. In addition, students often make mistakes in identifying the relationship between the value of the power and the position of numbers in complex fields. This leads to improper completion and misinterpretation of concepts. Therefore, the problem related to pattern recognition in complex number exponentials still needs to be studied more deeply.

A number of studies have shown that understanding complex numbers often focuses only on basic operations without paying special attention to the recognition of exponential patterns. Research Wakhata et al. (2023) emphasizing that students tend to prioritize the final outcome over the mathematical structure underlying the problem. However, these studies rarely examine the specific rotational patterns that arise in the exponential of complex numbers. In addition, there have not been many studies that have examined how students identify periodic patterns when faced with complex number ranks in the form of i, –1, –i, and 1. This gap is becoming increasingly apparent because most studies only observe procedural errors, not conceptual errors. According to Rizos & Gkrekas (2024) pattern recognition is an important cognitive aspect in mathematical thinking that is still underexplored. Thus, there is a real need to examine how students recognize patterns in the context of exponential complex numbers. Studies like this one will contribute to the development of a deeper understanding of concepts.

One alternative solution to overcome these problems is to strengthen students' ability to identify mathematical patterns. Pattern-based learning can improve students' cognitive capacity to understand mathematical structures (Zhang et al., 2026). Representational approaches can help students systematically understand the relationships between the ranks of complex numbers. In addition, the use of visualization through Argand diagrams can make it easier for students to observe repetitive rotation patterns. Haque (2024) argues that different forms of representation are essential for reducing cognitive barriers to learning mathematics. Teachers can also provide a series of exponential questions that highlight the regularity of the pattern so that students practice recognizing periodic structures. With this strategy, students have the opportunity to build their own understanding of exponential patterns. This approach is expected to help reduce conceptual errors when solving complex number problems.

Several previous studies have discussed students' difficulties in understanding complex numbers. Chang et al. (2024) finding that students often misinterpret imaginary concepts due to a lack of visualization experience. Santos-Trigo (2024) report that mathematical patterns play an important role in problem solving but are often not understood in depth by students. Other research by Olivares et al. (2021) showing that the ability to recognize patterns can predict students' success in solving high-level math problems. However, their research did not explicitly examine exponential patterns in complex numbers. Meanwhile, Jäder & Johansson (2025) highlights that abstraction of complex numbers often leads to misconceptions that interfere with problem-solving. Research Díaz-Berrios & Martínez-Planell (2022) It was also found that students' understanding of exponentiation was still limited to procedural manipulation. This suggests that there is still a special research space in the context of exponential patterns of complex numbers.

The uniqueness of this research lies in its main focus which examines the patterns that students recognize when solving complex number exponential problems. Previous research has highlighted more procedural errors, while this study emphasizes the cognitive aspect of pattern recognition. According to Schoenfeld (2020) patterns are at the heart of mathematical thinking, so the study of patterns on the topic of exponential complex numbers has become a new contribution to the literature. In addition, this study provides an in-depth analysis of the cyclical and representational patterns identified by students. Such studies have not been widely conducted in the context of complex numbers. This study also offers an analytical framework that combines cognitive approaches and error analysis. Thus, the novelty of this research lies in the exploration of the structure of patterns that appear in the students' thinking process. This study provides a new perspective that has not been addressed in research related to complex numbers.

Recent research shows a growing concern about the role of pattern recognition in mathematics learning. Demonty et al. (2018) state that patterning skills are the basis for high-level algebraic skills. In the context of complex numbers, recent studies have highlighted more the use of visual representations to support conceptual understanding. Some recent research such as Turgut (2022) shows that complex rotational visualizations can help students understand geometric transformations. However, there has been no research that specifically examines the exponential pattern of complex numbers from the perspective of students' thinking processes. These studies only discuss the representation aspect, not the ability to identify periodic patterns. Therefore, this research is in a state of the art position by combining pattern analysis, exponentiation, and student cognitive processes. This approach is relevant to the development of cutting-edge mathematics learning theory.

The urgency of this research is based on the need to improve students' understanding of complex numbers, particularly in the context of exponentiation. Difficulty in identifying patterns can lead to ongoing misconceptions. According to Chew & Cerbin (2021) misconceptions that are not addressed in the first place will hinder the formation of new, more correct cognitive structures. This shows that patterns need to be taught and researched systematically. In addition, complex numbers are the material that is the basis of various fields of engineering and science so that their understanding must be solid. Recognition of exponential patterns can help students solve more complex math problems in the future. This research is also important for designing learning that improves higher-level thinking skills. Thus, the urgency of this research is very strong from an academic and pedagogical perspective. Although several studies have examined pattern recognition in various mathematical contexts, there is limited research on how high school students recognize and use patterns in complex number rank operations, particularly in determining the cyclic shape of imaginary number powers and generalizing patterns to solve problems with large exponents. Therefore, this study aimed to explore the pattern recognition process used by grade XI students to solve complex number ranking problems.

This study aims to explore how Grade XI students recognize and utilize patterns when solving complex number exponentiation problems. Specifically, this study investigates the strategies students use to identify cyclic patterns and apply them to simplify high-order exponentiation problems. The findings are expected to contribute to a better understanding of students' mathematical reasoning and support the development of pattern-based learning approaches in mathematics education.

METHOD

Types and Approaches to Research

This study used qualitative research with an exploratory descriptive type. The qualitative approach was chosen because this study focuses on an in-depth exploration of students' thinking patterns and pattern recognition in solving complex exponential problems, especially how students identify cyclical patterns, apply rank reduction, and build a solution strategy. According to Mey (2023) qualitative research is appropriate when researchers want to understand cognitive processes naturally through students' work and explanations. Thus, this study does not aim to test hypotheses, but rather to describe cognitive phenomena in detail.

Subjects and Data Sources / Research Participants

The study subjects were three grade XI who were selected purposively based on their ability to solve complex number problems. The study involved three Grade XI students selected through purposive sampling. The selection was based on their ability to solve complex number problems and their willingness to articulate their reasoning during think-aloud sessions and interviews. The use of three participants is consistent with qualitative exploratory research, which prioritizes depth of understanding over statistical generalization. Each participant was treated as an information-rich case, allowing for an in-depth analysis of students' pattern-recognition processes, solution strategies, and mathematical reasoning. Therefore, the small number of participants was considered sufficient to achieve the study purpose. To capture diverse pattern recognition strategies, the participants were selected to represent different levels of mathematical performance. This approach enabled the researcher to obtain rich and varied qualitative data regarding students' reasoning processes in solving complex number exponentiation problems.

The study involved three Grade XI students selected through purposive sampling based on their ability to solve complex number problems and their willingness to explain their reasoning processes during think-aloud sessions and interviews. The selection of three participants was considered appropriate because the study adopted a qualitative exploratory approach that emphasizes depth of understanding rather than statistical generalization. Each participant was treated as an information-rich case, allowing the researcher to conduct an in-depth examination of students' pattern recognition processes, solution strategies, and mathematical reasoning. The combination of written work, think-aloud protocols, and semi-structured interviews generated sufficiently rich data to address the research objectives of this study. In qualitative research, sample adequacy is determined by the richness and relevance of the information obtained rather than by the number of participants. Therefore, the focus of this study was to obtain detailed insights into students' cognitive processes in recognizing patterns within complex number exponentiation problems. The selected participants provided comprehensive explanations and diverse solution strategies, enabling a thorough exploration of the phenomenon under investigation. The primary data sources consisted of students' worksheets (LKS) containing the solution to $(1+i)^{2026}$, records of students' thought processes obtained from the think-aloud sessions, and semi-structured interviews conducted to confirm the rationale underlying each solution step. Subject selection followed the recommendations of Campbell et al. (2020), who suggested that purposive sampling is appropriate when the objective of the research is to obtain in-depth information rather than statistical generalization of the results.

Data Collection Techniques

Data collection techniques included a problem-solving test in the form of one main question, $(1+i)^{2026}$, documentation of students' work, including scribbles, calculations, and pattern diagrams, a controlled think-aloud procedure in which students explained their thought processes while solving the problem, and semi-structured interviews conducted to validate the researcher's interpretation of each student's solution steps. The combination of these techniques provided data triangulation, in accordance with Schlunegger et al. (2024).

Research Instruments

The research instruments consisted of a test instrument, interview guidelines, and an observation and step analysis rubric. The test instrument consisted of one higher-order thinking skills (HOTS) question on exponential complex numbers: "*Calculate the value of* $(1+i)^{2026}$. *Include all steps and reason for calculation*." This question was designed to elicit students' ability to decompose complex forms, identify exponential patterns, apply the cyclic pattern of powers of i(mod 4), and generalize patterns for high exponents. The interview guidelines consisted of four main questions: why the student chose a particular solution step, how the student knew that the powers of $i$repeat every four, how the student determined the exponent reduction for 2026, and whether the student considered any alternative strategies while solving the problem. The researcher also employed an observation and step analysis rubric to evaluate students' solution processes. The rubric focused on pattern identification, calculation consistency, logical reasoning, and exponent reduction strategies.

Research Instruments

The research procedure consisted of four stages: research preparation, data collection, data analysis, and conclusion. During the research preparation stage, the researcher drafted the test instrument and interview guidelines, conducted expert validation of the research instrument, and selected the participants through purposive sampling. During the data collection stage, the problem was administered to each student individually. The researcher recorded each student's problem-solving process through written documentation and oral explanations while conducting think-aloud sessions and confirmation interviews. During the data analysis stage, students' written work was transferred into an analysis format. The researcher then conducted initial coding, axial coding, and selective coding to identify students' thinking patterns and conceptual errors. Finally, during the conclusion stage, the researcher compiled a description of each student's findings, organized the results into categories of strong, medium, and weak pattern recognition, and developed a descriptive model of students' cognitive structure in recognizing exponential patterns of complex numbers.

Data Analysis Techniques

Data analysis was conducted using an interactive model consisting of data reduction, data presentation, and conclusion drawing and verification. During the data reduction stage, the researcher compiled the complete record of each student's work and extracted important solution steps, including the expansion of $(1+i)^2$, simplification to $2i$, exponent reduction from $(1+i)^{2026}$to $(2i)^{1013}$, and the application of modulo 4. At this stage, conceptual errors and evidence of conceptual understanding were also identified.

During the data presentation stage, the researcher organized the data into a matrix of students' thinking patterns and constructed a process chart illustrating the sequence of students' reasoning, such as expansion → cycle → modulo 4 → generalization. Finally, during the conclusion drawing and verification stage, students' pattern recognition was classified into three categories: Type A, representing strong pattern recognition characterized by consistent reasoning and accurate generalization; Type B, representing partial pattern recognition in which students understood the cyclic pattern but made errors in exponent reduction; and Type C, representing weak pattern recognition characterized by misconceptions regarding exponents or complex number forms. The conclusions were then verified through triangulation with the interview data.

RESULTS

This study analyzes students' thinking patterns in solving complex exponential problems $(1+i)^{2026}$. An analysis was conducted against three student worksheets that showed the processes, strategies, and patterns used to identify the exponential relationships of complex forms. The three students showed a relatively similar procedure, namely expanding $(1+i)^2$, simplify it into $2i$, then reduce the rank and utilize the cyclic pattern of the rank of imaginary numbers (i repeat every four steps).

To improve data transparency and facilitate verification of the findings, excerpts of students' written solutions are presented alongside the figures. These excerpts highlight key stages of pattern recognition, including simplification of complex expressions, exponent reduction, identification of the cyclic pattern of i, and the application of modulo 4 reasoning. The excerpts enable readers to directly compare the students' written work with the interpretations provided in the analysis.

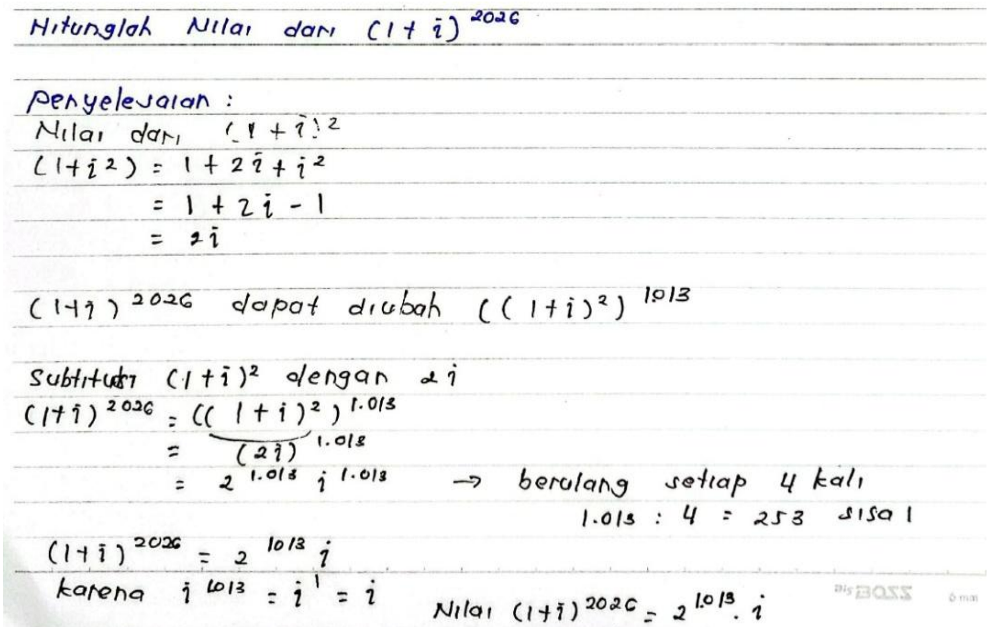

Figure 1. Student Work Results 1

Student 1 first simplified the complex expression, reduced the exponent using modulo 4 reasoning, identified the cyclic pattern of iii, and obtained the final solution correctly. This excerpt illustrates the student's ability to recognize and apply recurring patterns in complex number exponentiation.

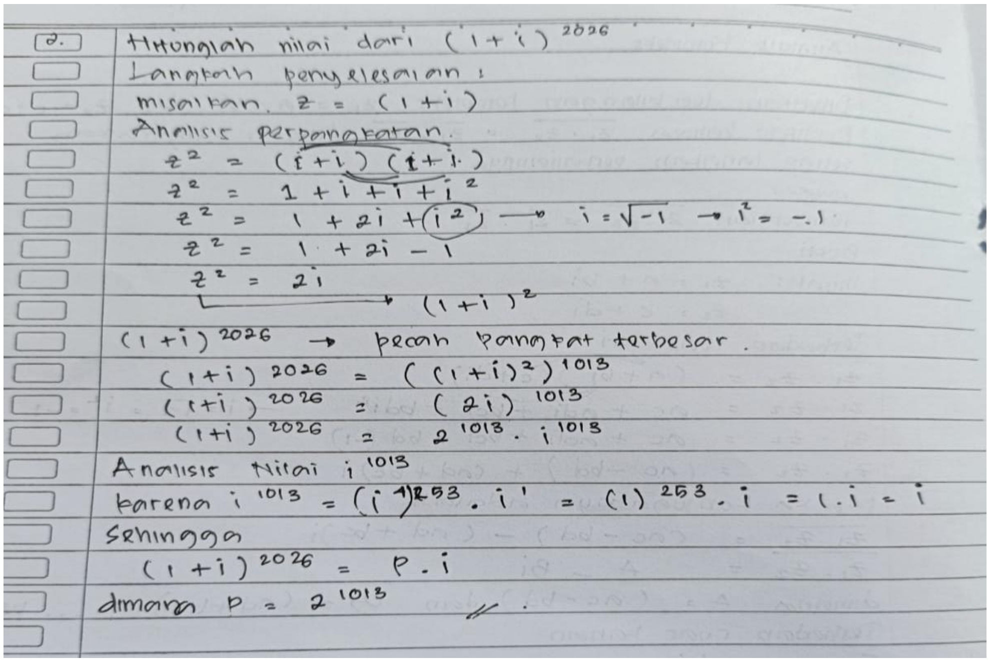

Figure 2. Student Work Results 2

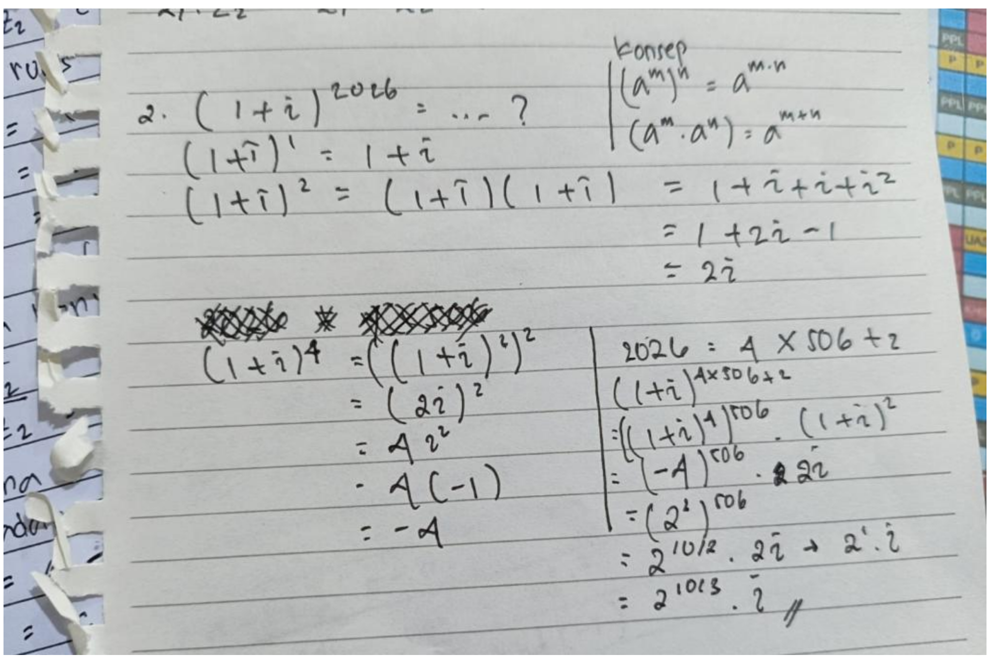

Figure 3. Student Work Results 3

Key Findings of Student Work

1. Student Work Findings 1

The results in Figure 1 show that Student 1 understood the exponential pattern of complex numbers through the following steps.

a. Counting $(1+i)^2 = 1 + 2i + i^2 = 2i$

b. Change rank: $(1+i)^{2026} = ((1+i)^2)^{1013} = (2i)^{1013}$

c. Outline: $(2i)^{1013} = 2^{1013}.i^{1013}$

d. Determining the cyclic pattern iii by division $1013 \div 4 \rightarrow sisa\ 1$
e. Concludes that $i^{1013} = i$
f. Final result: $(1+i)^2$=$2^{1013}.i$

Student 1 showed a very strong, consistent, and misconception-free pattern recognition. The steps are clean, systematic, and argumentative.

2. Student Work Findings 2

In Figure 2, Student 2 uses a similar approach but with several elaborative stages:

a. Expand $(1+i)^2 = 2i$
b. Announcing the concept of rank: $(a^m)^n = a^{mn}$
c. Compiling: $(1+i)^{2026} = ((1+i)^2)^{1013} = (2i)^{1013}$
d. Break down into: $2^{1013}.i^{1013}$
e. The cycle pattern was determined using 1013 ÷ 4 t
f. Summing up: $(1+i)^{2026} = 2^{1013}.i$

Student 2 understood the pattern exactly, but had a long and repetitive section of notes, suggesting that he needed stronger internal *scaffolding* despite being able to achieve the correct answer.

3. Student Work Findings 3

In Figure 3, Student 3 also shows an almost identical line of thinking to that of the other students:

a. Counting $(1+i)^2 = 2i$
b. Determine $(1+i)^4 = -4$ (additional flows that are not needed but still conceptually consistent)
c. Outline 2026 become 4 x 506 + 2
d. Rearrange: $(1+i)^{2026} = (1+i)^{4\cdot 506}.(1+i)^2$
e. Simplify: $(1+i)^{4\cdot 506} = (-4)^{506}$
f. Concluding the final form remains towards: $2^{1013}.i$

Student 3 shows creativity in strategy, namely using the division of ranks differently (using 4 × 506 + 2), but the results are still correct. This shows the ability to be strategically flexible.

General Pattern of Student Pattern Recognition

From the three students, the following general patterns were found.

1. All students start with identical steps : $\boldsymbol{(1+i)^2 = 2i}$
2. The three students made a major reduction in rank using the concept: $\boldsymbol{(1+i)^{2026} = ((1+i)^2)^{1013}}$
3. All three are aware of the cyclic pattern of rank $\boldsymbol{i}$: $\boldsymbol{i, -1, -i, 1}$ (repeat every 4)
4. All three use modular partitions: 1013÷ $\boldsymbol{4 = 253}$ sisa 1
5. All students concluded that: $\boldsymbol{i^{1013} = i}$
6. The end result of the whole subject is consistent: $\boldsymbol{(1+i)^{2026} = 2^{1013}.i}$

Table 1. Student Pattern Recognition Category

Students	Conceptual Precision	Strategy Consistency	Flexibility Strategy	Level Pattern Recognition
S1	Very precise	Very consistent	Medium	Height
S2	Precise	Consistent	Low	Medium–High
S3	Precise	Consistent	Height	Height

DISCUSSION

The results of the study showed that all students were able to solve exponential problems of complex numbers by utilizing patterns $(1+i)^2 = 2i$, which is then reduced to a form $(2i)^{1013}$. These findings indicate that students naturally use *pattern recognition* as a primary cognitive strategy. This is in accordance with the theory Zhang et al. (2018) which states that pattern recognition is a fundamental step in mathematical problem solving, as patterns allow for the simplification of complex problems into more understandable structures. All students were able to find cyclic patterns in the power of imaginary numbers, which repeated every four, thus shortening the computational process. These findings support the Hußmann & Prediger (2016) that learning is the process of finding structure, and the structure that students recognize in this study is an exponential cyclic pattern of complex numbers.

Further analysis showed that students not only recognized patterns, but were also able to relate them to the concepts of power and algebraic properties. For example, all students use the concept $(a^m)^n = a^{mn}$ to reduce the large rank to a simpler form. Understanding concepts in mathematics includes the ability to connect procedural and conceptual representations simultaneously (Borji et al., 2021; Özpınar & Arslan, 2022). In this context, students demonstrate an integration between calculation procedures and a conceptual understanding of exponential complex numbers. These findings show that students have a strong enough mental representation of complex number structures to be able to manage rank operations appropriately.

This study shows that there is a variation in strategies between students in reducing grades. Student 1 and Student 2 use the common form $(1+i)^{2026} = ((1+i)^2)^{1013}$, while Student 3 uses a different approach, namely breaking 2026 into 4×506+2. The variation in students' solution strategies indicates flexibility in mathematical thinking. Although the students used different approaches to reduce the exponent, they consistently arrived at the same correct solution (Elhilal, 2025; Hong et al., 2023; Verschaffel, 2024). This suggests that they understood the underlying mathematical structure rather than relying on a single memorized procedur. Flexibility like this is an important indicator of high level of mathematical literacy. Despite using different strategies, all three students still lead to the same outcome, which shows that they understand the underlying structure of the operation consistently.

The findings of the study also confirm that students are able to use the cyclic pattern of the power of imaginary numbers appropriately, especially in determining that $i^{1013} = i$, based on the remaining division of 1013 to 4. This process is in line with the modular arithmetic theory described by Schüler-Meyer (2019) that the repetition of the nature of the operation can be understood through the modulo arithmetic approach. This ability shows that students do not just memorize patterns, but use them to make mathematical inferences. This reflects the development of high-level thinking skills as stated by Changsri et al. (2024) that pattern recognition and predictive ability are key elements in *higher-order thinking*.

This study shows that pattern recognition is an essential component in solving the exponential problem of complex numbers. Students are able to demonstrate the integration between concepts, procedures, and patterns, which is characteristic of deep mathematical understanding as mentioned by (Cai & Ding, 2017; Schuchardt & Schunn, 2016). These findings also confirm the importance of providing questions that encourage students to recognize patterns and make generalizations. In addition, the results of the study imply that a learning approach that emphasizes *pattern-based reasoning* can improve students' ability to solve non-routine problems. Therefore, this research corroborates that understanding patterns is not only a strategy, but also a foundation in advanced mathematical thinking.

CONCLUSION

This study explored how Grade XI students recognize and utilize patterns when solving complex number exponentiation problems. The findings revealed that all participants were able to identify the cyclic pattern of powers of the imaginary unit and apply exponent reduction strategies using modulo arithmetic to simplify high-order exponentiation problems. Although the students employed different solution strategies, they consistently arrived at correct solutions, indicating a strong understanding of the underlying mathematical structure. These findings demonstrate that pattern recognition plays a crucial role in supporting students' mathematical reasoning, enabling them to connect conceptual understanding with efficient problem-solving strategies. Theoretically, this study contributes to the literature on pattern recognition in mathematics education by providing empirical evidence of how students use cyclic structures and modular reasoning when working with abstract mathematical concepts to solve mathematical problems. The findings suggest that pattern recognition functions as a cognitive bridge between conceptual knowledge and procedural fluency, particularly in complex number exponentiation. This contribution enriches current discussions regarding the role of pattern-based reasoning in the development of higher-order mathematical thinking. From a practical perspective, the findings imply that mathematics instruction should provide students with greater opportunities to explore patterns, formulate generalizations, and justify their reasoning processes. Learning activities that emphasize visualization, cyclic relationships, exponent reduction, and pattern-oriented problem solving may help students develop deeper conceptual understanding and reduce reliance on mechanical procedures. Therefore, integrating pattern recognition tasks into mathematics learning can support the development of more flexible and meaningful problem-solving strategies. This study is limited by the small number of participants, involving only three Grade XI students, which restricts the transferability of the findings to broader educational contexts. In addition, the analysis relied heavily on students' written work and verbal explanations during think-aloud sessions and interviews, which may have been influenced by individual differences in communication ability. Future research is recommended to involve a larger and more diverse group of participants representing different levels of mathematical ability. Further studies should investigate the effectiveness of instructional interventions based on pattern exploration, visualization, and mathematical reasoning in improving students' understanding of complex numbers and other advanced mathematical topics. Such studies would provide a stronger foundation for developing instructional models that foster pattern recognition as a core component of mathematical thinking.

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AUTHOR CONTRIBUTION STATEMENT

S contributed to conceptualization, methodology, supervision, validation, writing – original draft, and writing – review & editing. SM contributed to methodology, formal analysis, investigation, resources, and writing – review & editing. S and SM contributed to data curation, investigation, project administration, and writing – original draft. All authors have read and approved the final version of the manuscript.

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