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Retractive Thinking Characteristics of Mathematics Education Students in Solving Differential Equation Problems: Evidence from a Qualitative Case Study

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ABSTRACT

Purpose: This study aims to identify and describe the characteristics of students' retractive thinking in solving differential equation problems. The research focuses on how students recognize errors, evaluate solution procedures, and revise their mathematical reasoning during the problem-solving process. **Methods**: This qualitative study employed a case study design involving eight mathematics education students who had completed a Differential Equations course. Participants were selected through purposive sampling. Data were collected using think-aloud protocols, semi-structured interviews, and documentation of students' written work. Data were analyzed through data condensation, data display, and conclusion drawing, supported by triangulation and member checking to ensure credibility. **Findings**: The findings reveal three main characteristics of retractive thinking. First, students demonstrate *error recognition*, shown by their ability to identify inconsistencies in their solution steps during or after problem solving. Second, students exhibit *procedural re-examination*, where they return to earlier steps to verify formulas and computational processes. Third, students apply *solution revision strategies*, ranging from correcting algebraic errors to restructuring entire solution procedures. However, retractive thinking is not consistently supported by conceptual understanding, as many revisions remain procedural rather than conceptual. This indicates a gap between procedural fluency and conceptual awareness in differential equation problem solving. **Research Implications**: The findings suggest the importance of integrating metacognitive strategies in teaching differential equations, particularly activities that promote self-monitoring, error analysis, and reflective revision. These strategies can strengthen students' retractive thinking abilities. However, this study is limited by its small sample size and case study design, which restrict generalization. The findings are therefore context-bound and intended to provide analytical insight rather than statistical generalization. **Originality:** This study contributes to the literature by specifically identifying and describing retractive thinking characteristics in the context of differential equation problem solving, offering a process-oriented perspective beyond procedural error analysis

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INTRODUCTION

Differential equations are one of the fundamental topics in higher mathematics that describe relationships between a function and its derivatives. This topic is widely used to model dynamic phenomena in science, engineering, and social systems. However, solving differential equation problems requires not only procedural fluency but also strong conceptual understanding and metacognitive awareness (Kloeden, 2025). Students are expected to monitor, evaluate, and revise their thinking processes when errors occur during problem solving. In this study, the central construct is retractive thinking, defined as a metacognitive process in which learners intentionally return to previous solution steps, identify errors, evaluate the correctness of procedures, and reconstruct mathematical reasoning to produce a revised solution (Çini et al., 2023). This construct is closely related to metacognitive monitoring and self-correction processes in mathematical problem solving, where learners actively regulate their cognitive actions to improve accuracy and conceptual consistency.

Although differential equations are an important subject, many students have difficulty solving these types of problems. These difficulties are not only related to procedural skills, but also reflective abilities in overcoming errors. The ability to reflect on wrong steps is an important foundation in solving mathematical problems (Verschaffel et al., 2020). However, the reality is that many students are unaware of the mistakes they make during the completion process. This difficulty causes students to often make repetitive steps without improvement. Additionally, they are less likely to have a strategy to independently identify the source of error. This creates problems in the learning process because the lack of understanding of concepts cannot be corrected. This problem becomes even more serious when students are unable to relate basic concepts to the solution method. Thus, there is a need to specifically examine how students react to solving differential equation problems.

Research on the difficulty of learning differential equations has been conducted, but studies on reactive thinking are still very limited. Many studies emphasize more on procedural skills without examining the mental processes that occur when students improve their steps. High-level thinking analysis including revision and evaluation is an important aspect of cognitive taxonomy (Damayanti et al., 2020; List & Sun, 2023; Radmehr & Drake, 2019). However, most studies only evaluate the ability of the end result, not the reactive thinking process that occurs. This gap leads to a lack of a deep understanding of how students recognize and correct mistakes. In addition, previous research has not revealed a model of the characteristics of reactive thinking in detail in the context of advanced mathematics material. This gap shows the importance of a study that looks at students procedurally, not just from the results of the right or wrong questions. Thus, there are research gaps that must be filled to further understand the dynamics of student reactive thinking.

To overcome this gap, a research approach is needed that can explore students' thinking processes in depth. The qualitative approach is one of the right choices because it is able to capture the phenomenon of thinking naturally. Qualitative research allows researchers to gain a richer understanding of individual cognitive processes (Bhattacharya, 2017; Walther et al., 2017). With this method, researchers can trace how students identify errors and correct them. This gap is important because differential equations require a combination of procedural accuracy and conceptual understanding. Without the ability to critically re-evaluate their solution steps, students may repeatedly reproduce errors without realizing their conceptual misunderstandings. Therefore, investigating retractive thinking can provide deeper insight into how students regulate and improve their mathematical reasoning during problem solving.

Several previous studies have examined students' thinking processes in various mathematical contexts. The process of reflective thinking has a strong influence on the understanding of mathematical concepts (Demirel et al., 2015; Tisngati & Genarsih, 2021). Furthermore, the research Thakral et al. (2023) emphasizing the importance of error correction in increasing thinking flexibility. Other research by Lee et al. (2023) emphasized that cognitive scaffolding helps students in correcting thinking errors. Meanwhile, research by Melker et al. (2025) reveals that dual representation can facilitate the process of strategy revision. Research by Izzati & Mahmudi (2018) it shows that mathematical problem-solving is not only a procedural skill, but is strongly influenced by metacognitive processes. However, from the study, no special focus has been found on reactive thinking in the context of differential equations. Therefore, this research fills this gap. By referring to these five sources, this study strengthens the urgency of the study of reactive thinking.

The uniqueness of this research lies in its focus on identifying the characteristics of student reactive thinking in detail in the context of differential equations. Most previous studies have only highlighted errors or settlement strategies without examining the revisional process. This research offers a different approach by tracing the student's thought trail from start to finish. The ability to monitor and control the thought process is a core part of the error correction mechanism (Bascandziev, 2024; Ghanizadeh, 2017). Thus, this research makes a new contribution in examining the metacognitive dimension in advanced mathematics materials. In addition, this study develops a new category regarding the forms of reactive thinking of students. The findings are expected to enrich the literature on student cognition in solving mathematical problems. This novelty is important because it has the potential to be the basis for the development of cognitive assessment instruments in the future. Thus, this study presents a new perspective in the study of differential equations.

Error detection and correction are also recognized as essential components of mathematical thinking. Research on cognitive processes indicates that learners often experience difficulties not only in executing procedures but also in recognizing and correcting their own mistakes. In addition, studies on higher-order thinking emphasize that revision processes and self-evaluation are critical indicators of advanced cognitive engagement in mathematics learning Despite these findings, research that specifically examines retractive thinking in the context of differential equations remains limited. Most existing studies focus on either procedural performance, general metacognitive ability, or error analysis separately, without explicitly investigating how students return to previous steps and reconstruct their reasoning during problem solving. Moreover, studies that integrate process-oriented analysis of student thinking in advanced mathematics topics are still scarce. This state of the art research refers to the latest developments regarding the study of mathematical thinking and metacognition. Recent literature shows that cognitive research in mathematics is moving towards process analysis, not just the final outcome. The thought process is the core of mathematical understanding (Goos & Kaya, 2020; Jablonka, 2020).

The urgency of this research lies in the need to understand students' thinking processes to improve the quality of mathematics learning. When students are able to identify and correct mistakes, they will achieve a deeper understanding. This is in accordance with the view that meaningful learning depends on the ability to revise knowledge (Bryce & Blown, 2024; Fiorella & Zhang, 2018). In practice, many college students do not have adequate metacognitive strategies. This condition leads to continued difficulties in high-level math material. By understanding the characteristics of reactive thinking, teachers can design more effective learning interventions. In addition, the urgency of this research is related to the need for the world of education to develop a metacognition-based learning model. Without a deep understanding of students' thinking processes, learning strategies are difficult to optimize. Thus, this research is very urgent to be conducted.

Based on this background, this study aims to identify and describe the characteristics of students' retractive thinking in solving differential equation problems. Specifically, it explores how students recognize errors, evaluate solution processes, and revise their mathematical reasoning. By focusing on these processes, this study contributes to a deeper understanding of metacognitive regulation in advanced mathematics learning and provides implications for instructional design that supports reflective and self-corrective learning strategies.

METHOD

Types and Approaches to Research

This study uses a qualitative approach with a case study design that aims to explore in depth the characteristics of retractive thinking of students in solving differential equation problems. This approach was chosen because it is suitable for tracing thought processes that cannot be observed only from the final result of the work. Case studies allow researchers to understand phenomena in detail through various data sources (Voss et al., 2016). This research focuses on the analysis of students' thinking processes when realizing, evaluating, and correcting mistakes during problem solving.

Research Design

This study employed a qualitative single-case study design. The case was bounded by one Mathematics Education Study Program at a public university in Indonesia, involving students who had completed the Differential Equations course during the 2024/2025 academic year. The study focused exclusively on understanding how retractive thinking emerged during mathematical problem solving within this specific instructional context. Thus, the unit of analysis was the cognitive process of individual students embedded within one educational setting.

Research Subject

The participants consisted of eight undergraduate Mathematics Education students who had completed the Differential Equations course. Participants were selected using purposive sampling based on three criteria: (1) they had completed the Differential Equations course, (2) they demonstrated adequate mathematical communication skills, and (3) they were willing to participate in think-aloud and semi-structured interview sessions. Mathematical communication ability was assessed by the lecturer who taught the Differential Equations course using three indicators: (1) clarity in explaining mathematical reasoning during classroom discussions, (2) quality of written mathematical explanations in coursework, and (3) ability to verbalize problem-solving processes during a pilot think-aloud activity conducted before the main data collection. The sample size of eight participants was determined based on the principle of information power (Vasileiou et al., 2018). Data collection continued until no substantially new characteristics of retractive thinking emerged during the analysis. The final participant did not contribute additional categories or patterns beyond those identified previously, indicating that sufficient information power and pattern saturation had been achieved. All participants were anonymized using codes (Subjek 1–Subjek 8) to maintain confidentiality.

Data Type

The data collected in this study consisted of three types. First, thinking process data were obtained through think-aloud recordings while students solved differential equation problems, enabling the researcher to capture participants' cognitive processes during problem solving. Second, data on students' work results were collected in the form of written answer sheets, which were analyzed to identify error patterns and the revision processes undertaken by the students. Third, in-depth interview data were gathered to explore the underlying reasons, problem-solving strategies, and retractive mechanisms that emerged throughout the students' solution processes.

Data Collection Techniques

Data were collected using three complementary techniques. First, the think-aloud method was employed, in which students solved differential equation problems while verbalizing their thought processes to provide insight into their cognitive activities during problem solving. Second, semi-structured interviews were conducted after the problem-solving session to clarify students' reasoning, decision-making processes, and revision strategies that could not be fully captured through verbal protocols alone. Third, documentation was used to support the analysis by collecting students' written answer sheets, audio recordings of the think-aloud sessions, and interview transcripts.

Research Instruments

The research instrument consisted of six open-ended problems designed to explore students' retractive thinking processes while solving differential equation problems. The instrument included two prerequisite calculus problems covering derivatives and integrals, two ordinary differential equation (ODE) solution problems, one classification task requiring students to distinguish between ordinary differential equations (ODE) and partial differential equations (PDE), and one modelling problem in which students were required to formulate and solve a differential equation based on a given situation. These tasks were designed to encourage authentic mathematical reasoning rather than intentionally induce errors, allowing errors to emerge naturally from students' conceptual understanding and problem-solving processes. The detailed specifications of the differential equation problem set, including the objective and cognitive target of each problem, are presented in Table 1.

Table 1. Specifications of the Differential Equation Problem Set

Problem	Objective	Cognitive Target
1	Derivative	procedural monitoring
2	Integral	error detection
3	Solve ODE	strategy revision
4	Solve ODE	conceptual revision
5	ODE vs PDE	conceptual monitoring
6	Modelling	full retractive thinking cycle

The instruments were validated by three experts consisting of two mathematics education professors specializing in mathematical cognition and one qualitative research expert. Validation focused on content relevance, construct alignment, language clarity, and suitability for eliciting retractive thinking. Suggestions from the validators were incorporated before data collection. To ensure consistent interpretation during qualitative analysis, a coding framework was developed to classify the observed forms of retractive thinking. The operational definitions of each analytical code are presented in Table 2.

Table 2. Codebook for Retractive Thinking Analysis

Code	Operational Definition	Inclusion	Exclusion
ER	Error Recognition	student realizes error	guessing
PR	Procedural Re-examination	checks previous steps	reading question
SR	Solution Revision	changes strategy	minor rewriting
CR	Conceptual Reconstruction	changes underlying concept	

Data Analysis Techniques

Data analysis was conducted using the interactive model consisting of data condensation, data display, and conclusion drawing and verification. During the data condensation stage, transcripts from the think-aloud sessions and semi-structured interviews were coded to identify patterns of retractive thinking. The coded data were then organized into matrices to display the relationships among error types, awareness processes, and revision strategies, facilitating systematic interpretation. In the conclusion drawing and verification stage, the findings were interpreted to construct categories of retractive thinking and were validated through triangulation and member checking to enhance the credibility of the results. A concurrent think-aloud protocol was employed throughout the data collection process. Prior to the formal sessions, each participant completed a 10-minute practice session to become familiar with verbalizing their thoughts while solving problems. When a participant remained silent for more than 10 seconds, the researcher provided a neutral prompt to encourage continued verbalization without directing the participant's reasoning. Each data collection session lasted approximately 60–75 minutes, consisting of approximately 45 minutes of problem solving followed by a 20-minute semi-structured interview. All sessions were audio- and video-recorded, and the transcripts preserved pauses, hesitations, corrections, erased writing, gestures indicating revisions, and the chronological development of written solutions. The semi-structured interviews were conducted to confirm participants' reasons for recognizing errors, revisiting previous solution steps, selecting alternative strategies, and reconstructing the mathematical reasoning that emerged during the think-aloud sessions.

Research Procedure

Prior to the main study, all prospective participants completed a 10–15 minute pilot think-aloud session using a mathematical task different from the research instrument. The session was intended solely to familiarize participants with verbalization procedures and to assess their ability to articulate their mathematical reasoning. Data from the pilot session were not included in the analysis. The research was conducted in four stages:

1. Preparation of instruments and validation of problem sets
2. Implementation of think-aloud-based problem solving sessions
3. Data collection through interviews and documentation
4. Data analysis and formulation of retractive thinking characteristics

RESULTS

Proses Retractive Thinking

The results are presented based on triangulated data from think-aloud protocols, semi-structured interviews, and students' written work (Figures 1–8). The analysis focuses on identifying patterns of retractive thinking, namely how students recognize errors, re-examine procedures, and revise their mathematical reasoning during differential equation problem solving.

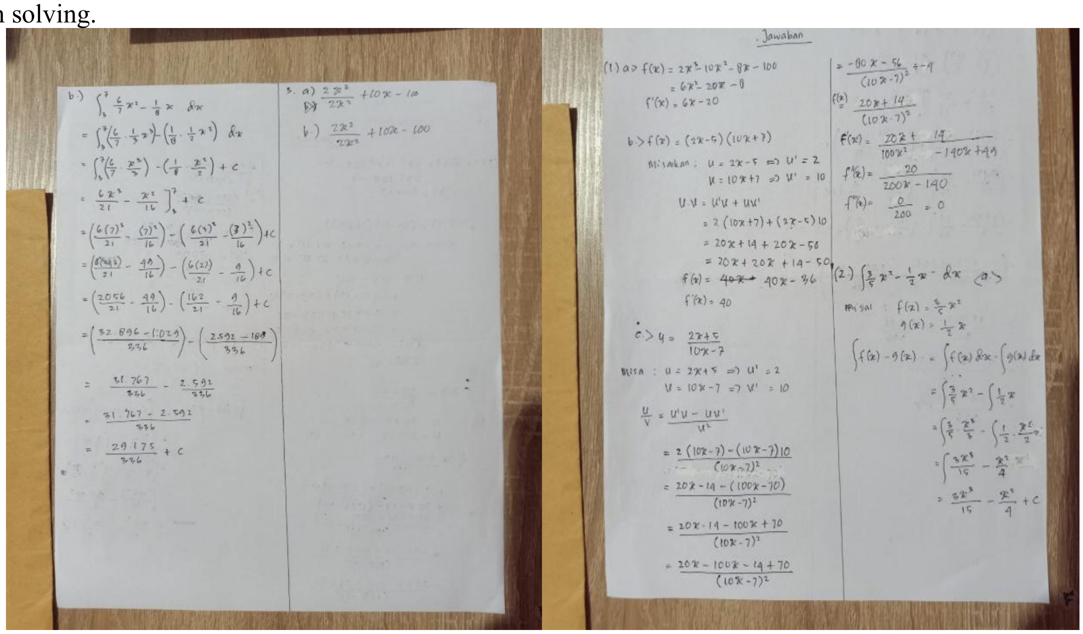


Figure 1. Subject 1

Student incorrectly applied the quotient rule. Think-aloud excerpt

"Wait... I think I used the wrong formula. The denominator should stay squared. Let me go back."

Interview excerpt

"I realized my mistake after comparing this step with the product rule that I learned before."

Researcher's interpretation

This sequence indicates spontaneous Error Recognition, followed by Procedural Re-examination, and finally Solution Revision.

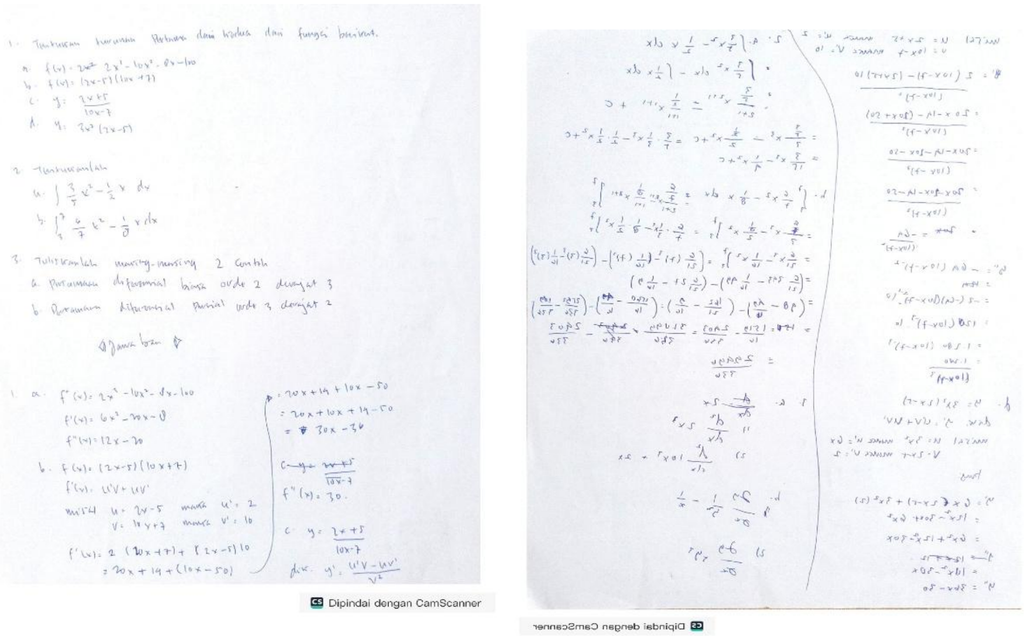


Figure 2. Subject 2

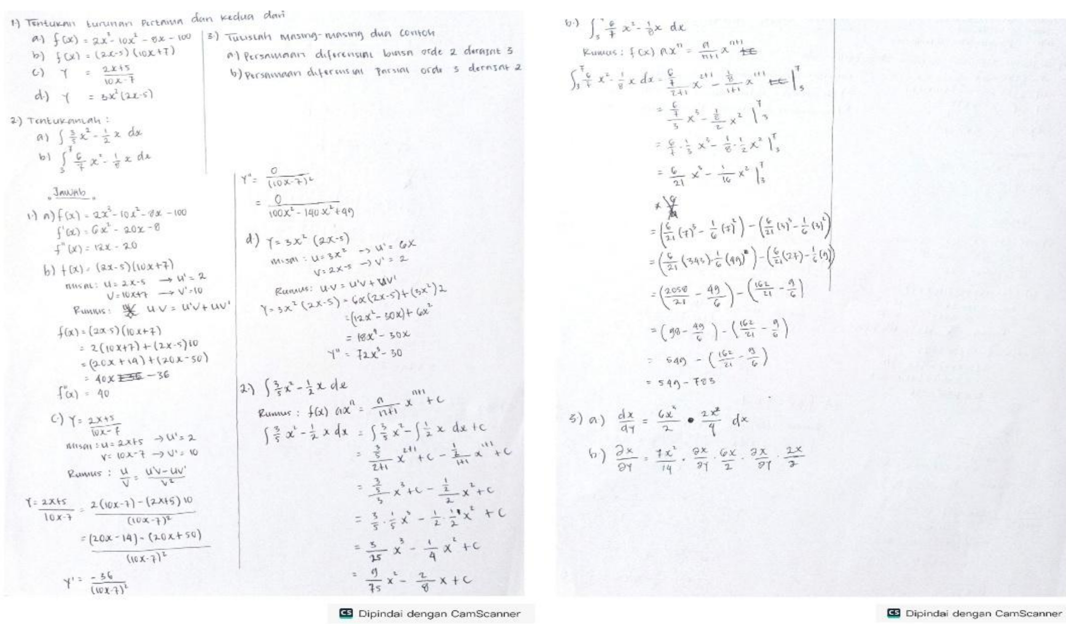


Figure 3. Subject 3

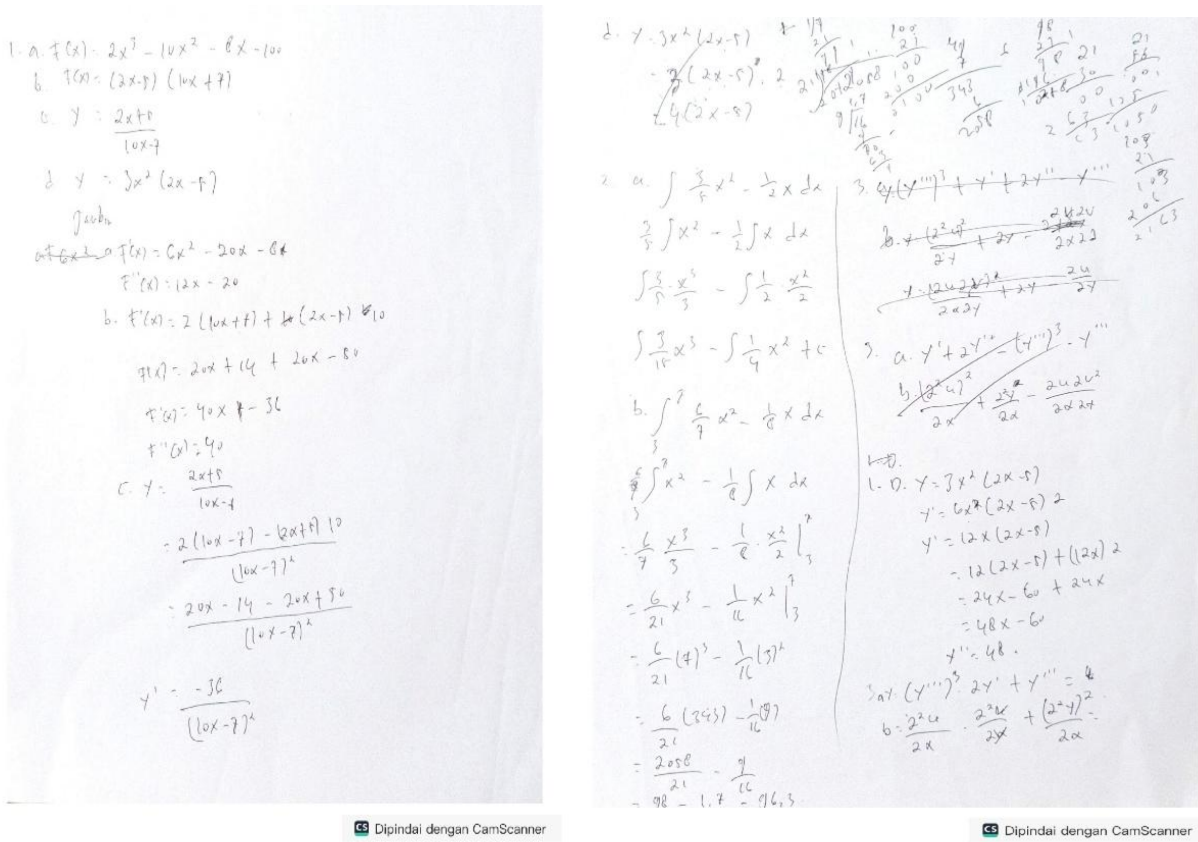


Figure 4. Subject 4

Student crossed out the denominator.Think-Aloud

"No...this cannot be correct."

Interview

"I remembered the quotient rule after checking my previous calculation."

Interpretation

The participant demonstrated spontaneous error recognition before revising the procedure.

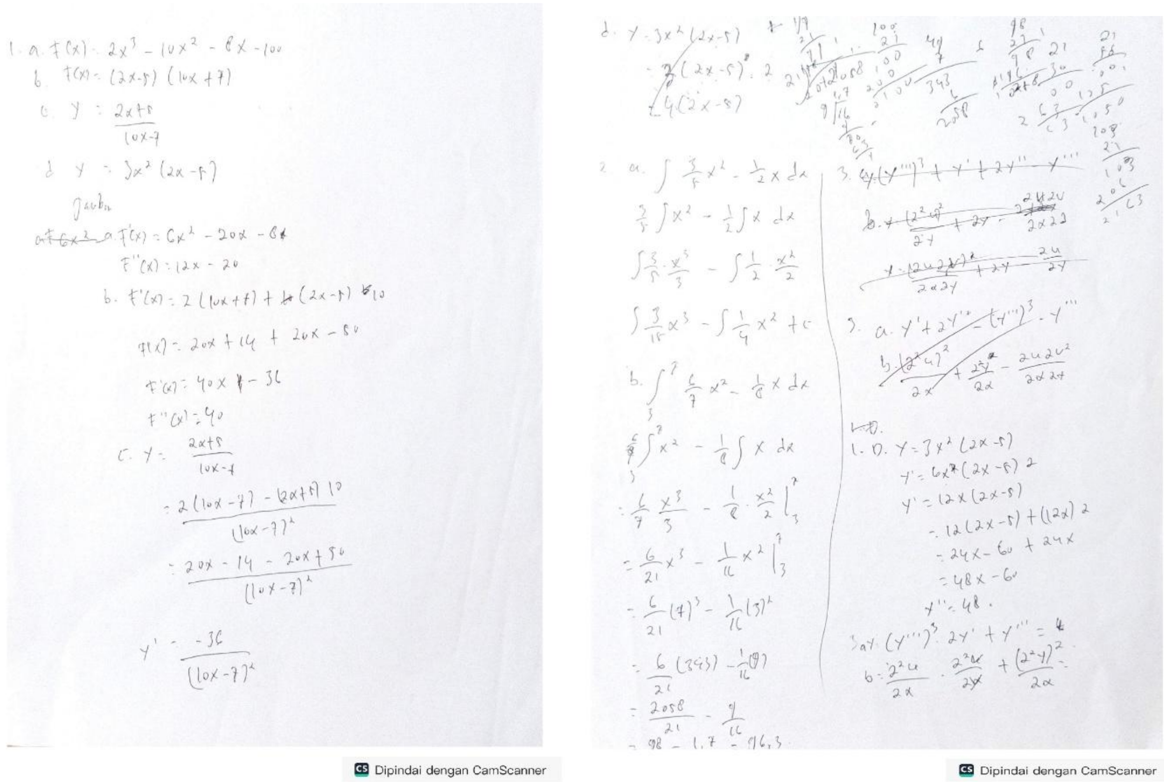
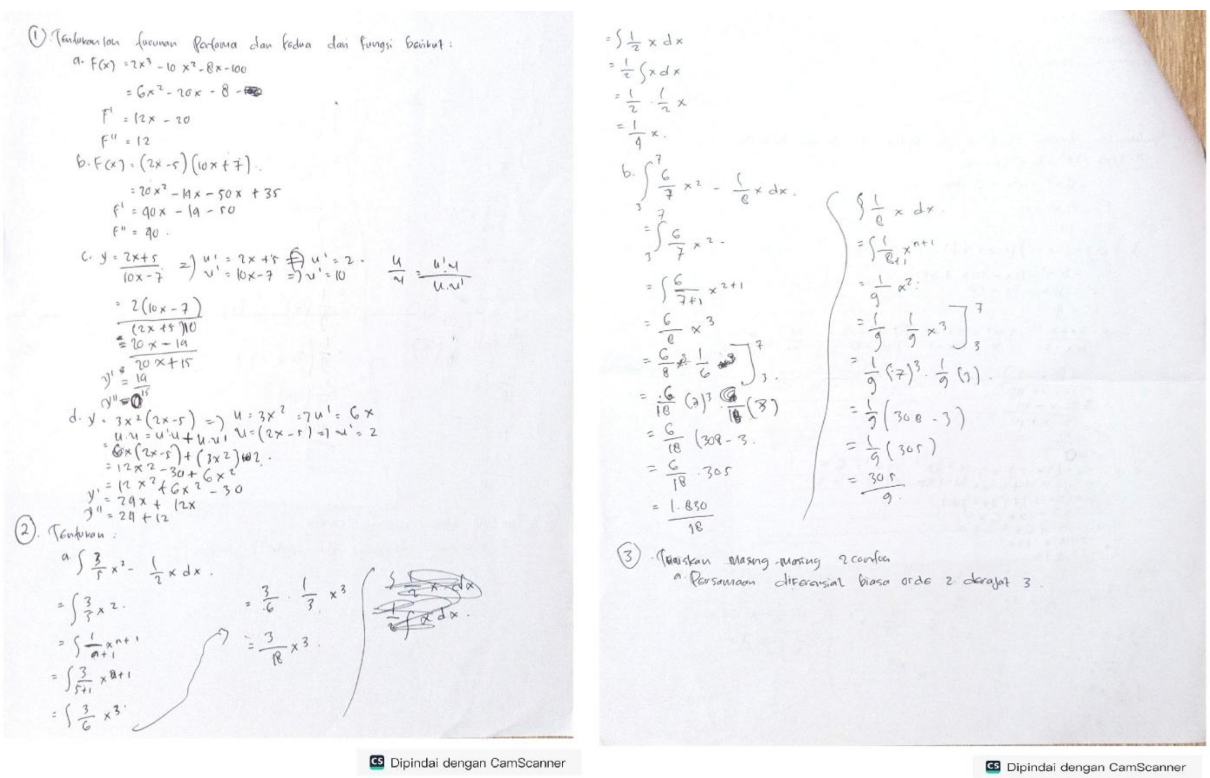


Figure 5. Subject 5



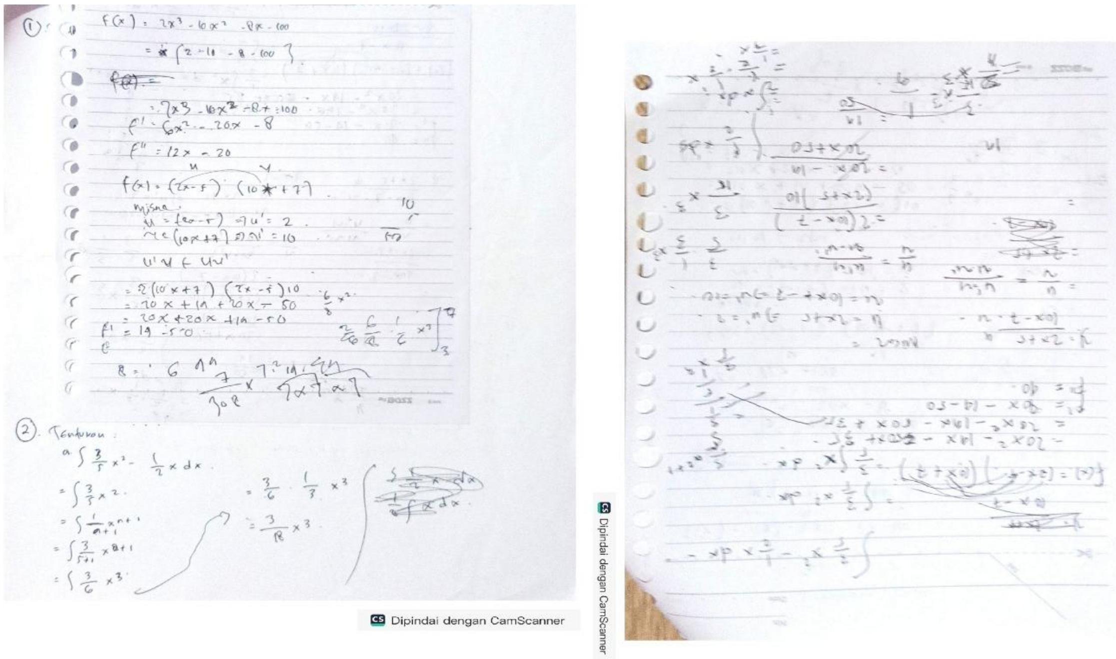


Figure 6. Subject 6

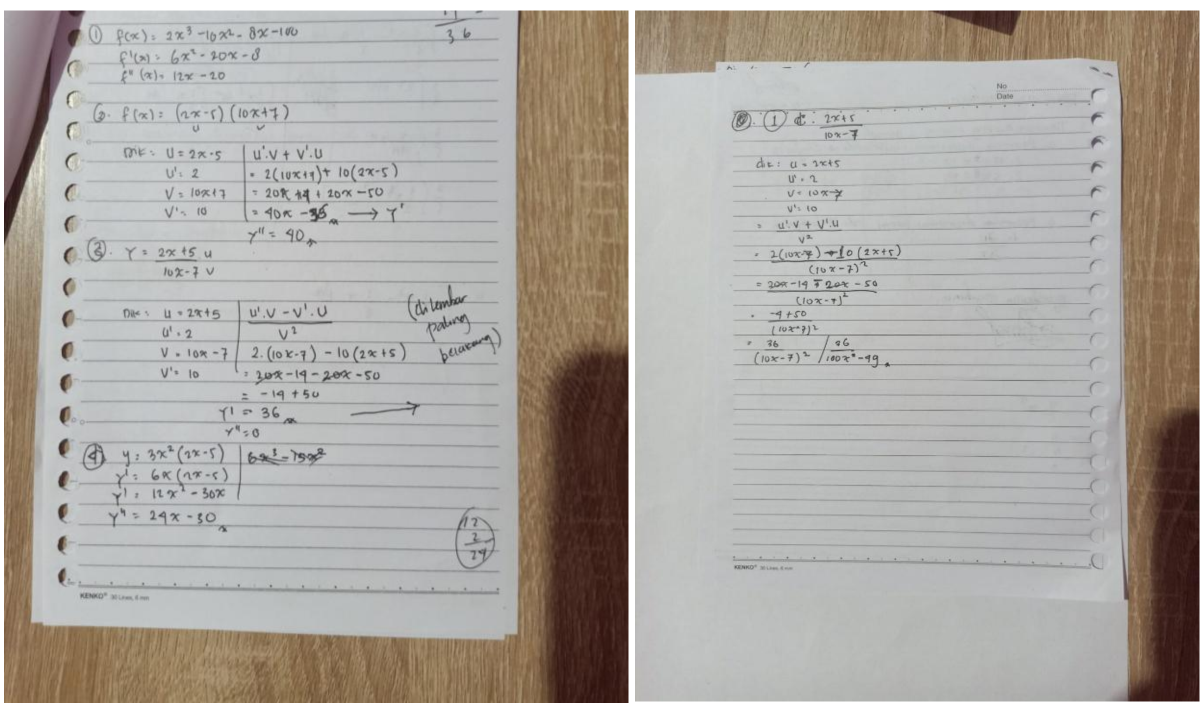


Figure 7. Subject 7

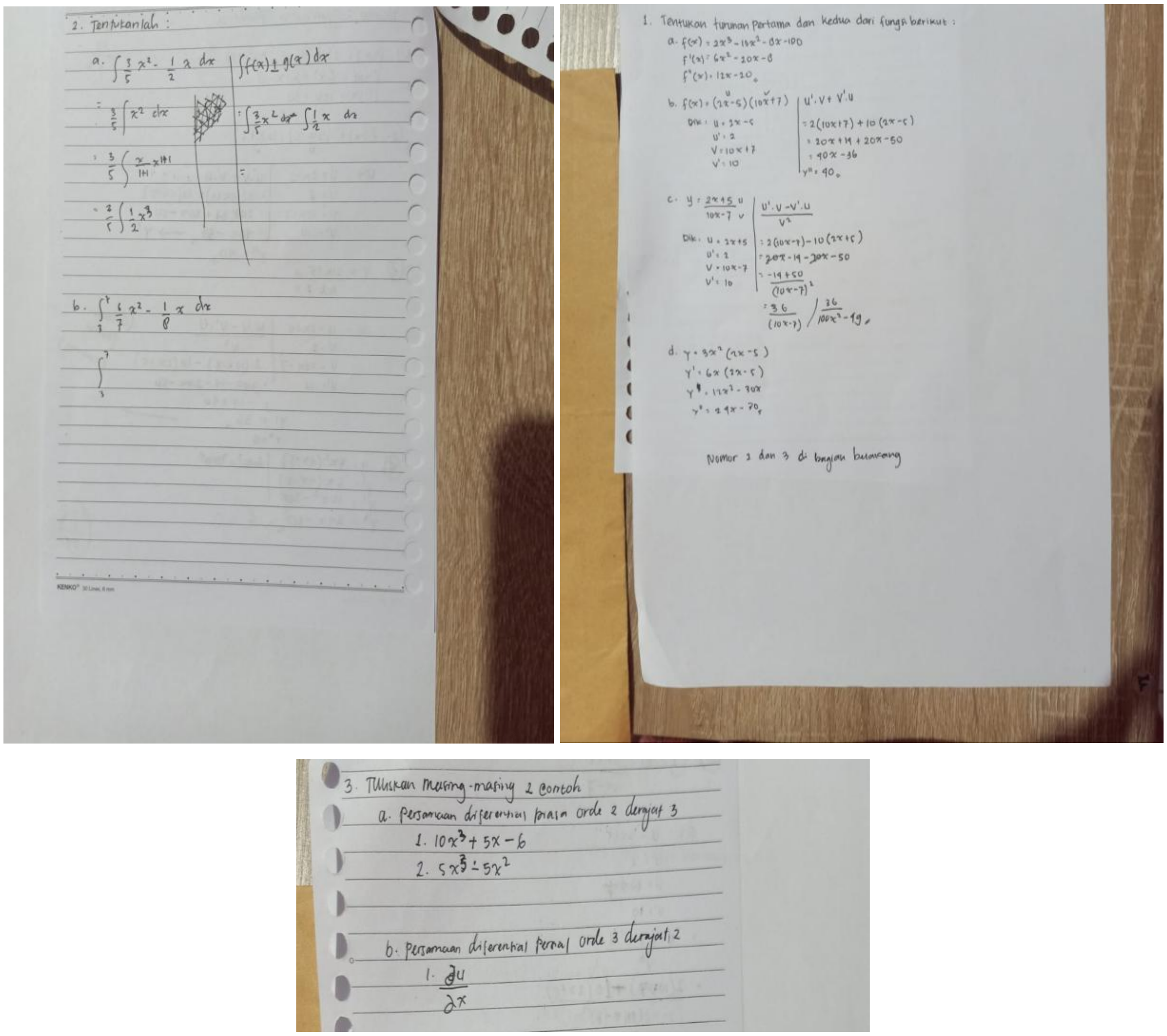


Figure 8. Subjek 8

Table 3. Matrix of Retractive Thinking Across Participants

Participant	Task	Initial Error	Error Recognition	Procedural Re-examination	Revision	Evidence
S1	Quotient Rule	Wrong formula	spontaneous	checked previous formula	procedural revision	think-aloud + worksheet
S2	ODE	wrong substitution	after mismatch	recalculated	strategic revision	worksheet + interview
S3	PDE	conceptual misunderstanding	after interview	reread definition	conceptual reconstruction	interview

Table 4. Participant Classification

Category	Number of Participants
Recognized and successfully revised errors	5
Recognized errors but failed to revise appropriately	2
Failed to recognize errors	1

Tables 3 and 4 summarize the overall findings obtained from the triangulation of think-aloud protocols, interview data, and students' written work. Table 3 presents a comparative matrix of retractive thinking across participants by describing the initial errors, the processes of error recognition, procedural re-examination, and solution revision, together with the supporting evidence. Based on these observed patterns, Table 4 classifies participants according to their ability to recognize and revise errors. These summaries provide an overview of the variation in students' retractive thinking before the findings are discussed in greater detail according to each stage of the retractive thinking process.

Overview of Students' Solution Patterns (Figures 1–8)

Analysis of Figures 1–8 shows that students' solution processes vary significantly across tasks involving derivatives, integrals, and differential equation classification. These figures serve as the primary empirical evidence illustrating students' initial solution construction, the occurrence of errors, and their subsequent revision attempts. Across all participants, a consistent pattern emerges in which students demonstrate stronger procedural fluency when solving routine derivative problems but encounter greater difficulty when faced with structural changes in formulas, particularly in the quotient rule and differential equation classification tasks.

Core Characteristics of Retractive Thinking

Part A: Error Recognition

Students were asked to determine the first derivative $\boldsymbol{f'(x)}$and the second derivative $\boldsymbol{f''(x)}$of several types of functions. For the polynomial function $\boldsymbol{2x^3 - 10x^2 - 8x - 100}$, the majority of students (Subjek 3, Subjek 8, Subjek 2, Subjek 5, and Subjek 4) solved the problem correctly through the second derivative, obtaining the final result $\boldsymbol{12x - 20}$. However, Subjek 1 exhibited inconsistency in mathematical notation by writing the derivative result as $\boldsymbol{6x - 20}$and placing the notation $\boldsymbol{f'(x)}$at a confusing stage of the solution process.

In the product rule problem $(2x-5)(10x+7)$, almost all students demonstrated procedural understanding of the differentiation formula $u'v + uv'$. Most participants successfully obtained the correct first derivative, $40x - 36$, indicating that the product rule had been internalized as a procedural algorithm.

A different pattern emerged in the quotient rule problem $y = \frac{2x+5}{10x+7}$, which clearly distinguished students with conceptual understanding from those relying primarily on memorized procedures. Subjek 2 and Subjek 4 correctly applied the quotient rule formula $\frac{u'v-uv'}{v^2}$, leading to the correct derivative $y' = \frac{-36}{(10x-7)^2}$. In contrast, Subjek 1 displayed a fundamental misconception by incorrectly substituting variables into the formula before abandoning the quotient rule altogether. The student instead differentiated the numerator and denominator separately, effectively using the invalid expression $\frac{u'}{v'}$, which produced the incorrect derivative $f''(x) = \frac{20}{200x-140}$. In addition, Data Document 7 revealed an algebraic error in expanding the squared binomial, where $(10x-7)^2$was incorrectly simplified to $100x^2 - 49$.

Part B: Procedural Re-Examination

For the indefinite integral problem $\int \left(\frac{3}{5}x^2 - \frac{1}{2}x\right) dx$, all participants demonstrated an understanding of the basic integration rules. Although some students, such as Subjek 1 and Subjek 2, expressed their final answers in the unsimplified form $\frac{3}{15}x^3 - \frac{1}{4}x^2 + C$, others simplified the coefficient directly to obtain $\frac{1}{5}x^3 - \frac{1}{4}x^2 + C$. These differences reflected variation in simplification strategies rather than conceptual misunderstanding.

A different pattern appeared in the definite integral problem $\int_3^7 \left(\frac{6}{7}x^2 - \frac{1}{8}x\right) dx$. Solving this problem required considerable computational precision because simplifying the fractional expressions involved a common denominator of 336. Data Document 2 recorded students' detailed arithmetic processes in obtaining the final value $\frac{29175}{336}$. The primary source of difficulty in this task was not the calculus concept itself but maintaining numerical accuracy throughout the calculations. An additional procedural error was observed when several students continued to include the integration constant $+C$in the final solution of a definite integral.

Part C: Solution Revision
Students were instructed to construct their own examples of a second-order, third-degree Ordinary Differential Equation (ODE) and a third-order, second-degree Partial Differential Equation (PDE). This task produced the highest level of difficulty among all assessment sections and revealed substantial conceptual misconceptions. Subjek 8, for example, wrote the algebraic expression $10x^3 + 5x - 6$and considered it to be an ordinary differential equation despite the complete absence of differential notation such as $d$or ∂. Other participants, including Subjek 2 and Subjek 3, attempted to include differential symbols such as $\frac{d^2y}{dx^2}$or $\frac{\partial\mu}{\partial x}$. Nevertheless, the resulting equations failed to satisfy the required order and degree specified in the problem, indicating incomplete understanding of the defining structural characteristics of differential equations.

Error Distribution Across Topics

Students were instructed to construct their own examples of a second-order, third-degree Ordinary Differential Equation (ODE) and a third-order, second-degree Partial Differential Equation (PDE). This task produced the highest level of difficulty among all assessment sections and revealed substantial conceptual misconceptions. Subjek 8, for example, wrote the algebraic expression $10x^3 + 5x - 6$and considered it to be an ordinary differential equation despite the complete absence of differential notation such as $d$or ∂. Other participants, including Subjek 2 and Subjek 3, attempted to include differential symbols such as $\frac{d^2y}{dx^2}$or $\frac{\partial\mu}{\partial x}$. Nevertheless, the resulting equations failed to satisfy the required order and degree specified in the problem, indicating incomplete understanding of the defining structural characteristics of differential equations.

Integrated Interpretation of Figures 1–8

Figures 1–8 collectively demonstrate that retractive thinking develops as a non-linear cognitive process. Students typically begin with an initial procedural execution, followed by either successful or unsuccessful detection of errors, partial or complete re-evaluation of intermediate solution steps, and eventual revision of their solutions with varying levels of conceptual depth. The findings further indicate that not all students complete the full retractive thinking cycle, as some remain at the level of procedural correction without progressing toward deeper conceptual reconstruction.

Ethical and Data Integrity Statement

All participants are anonymized as Subjek 1–Subjek 8. No personal identifiers are used. Figures 1–8 are included as essential empirical data supporting the validity of the analysis and must be retained in the final manuscript for verification and reproducibility of the findings.

To facilitate the interpretation of the findings, the characteristics of retractive thinking identified during the analysis were organized into three complementary analytical categories. Table 5 summarizes the levels of error recognition observed among participants, Table 6 presents the categories of procedural re-examination undertaken during problem solving, and Table 7 describes the types of solution revision, ranging from surface-level procedural corrections to conceptual reconstruction. Together, these analytical categories provide a structured framework for understanding the progression of students' retractive thinking throughout the problem-solving process.

Table 5. Error Recognition

Level	Description
ER1	spontaneous recognition
ER2	recognition after computational inconsistency
ER3	recognition after interview prompt

Table 6. Procedural Re-examination

Code	Description
PR1	formula checking
PR2	algebra checking
PR3	substitution checking
PR4	definition checking

Table 7. Solution Revision

Code	Description
SR1	surface revision
SR2	procedural revision
SR3	strategic revision
CR	conceptual reconstruction

DISCUSSION

The results of this study show that five of the eight participants have mastered basic procedural skills in solving the derivative operations of polynomial functions. Research subjects such as (Subjek 3, Subjek 8, Subjek 2, Subjek 5, Subjek 4) showed very stable performance when determining the first and second derivatives of the three-power function. This mechanical success indicates that the process of working on basic rank rules has been well stored in their long-term memory. This phenomenon of high procedural ability is in line with the theory of instrumental understanding initiated by Sweller (2016). Instrumental understanding is the ability to apply ready-made rules or formulas to solve a problem without knowing the logical reason behind it (Österman & Bråting, 2019). Based on the results of the exam, students immediately multiply the rank of the variable in the future and subtract it by the number one automatically without conceptual barriers. This dominant procedural skill helps students solve initial questions quickly and minimize mistakes in writing mathematical symbols. Thus, the basic capital of calculating student calculus at the differential level of independent polynomial functions is already in a very satisfactory category.

When moving on to testing the multiplication rules of functions, the level of accuracy of students is still at a well acceptable level. The application of the two-variable multiplication formula has been proven to be successfully executed by the majority of subjects to produce a consistent final value. However, a very drastic decline in performance began to be seen when students were faced with the rules of rational division of functions. The structure of the division formula involving the subtraction and the square of the denominator demands a much stricter level of procedural compliance. This cognitive barrier triggered the birth of various mathematical misconceptions made by some of the respondent students during the exam (Özdemir & Toker, 2025; Schneider, 2020). One of the most striking misconceptions was found in Siti La Misran's answer sheet. He ignored the standard formula for the division of functions and chose to descend the numerator and denominator separately independently. The failure to process this rational function proves the existence of a serious weak point when the general procedure undergoes a shape modification.

The phenomenon of algebraic operation errors in the division rule can be dissected in depth using the framework of Error Analysis Theory (Mathaba et al., 2024). Five hierarchical stages that trigger student errors in mathematics, namely reading, understanding, transformation, process skills, and answer writing (Siregar et al., 2018). If referring to the results of the research, the biggest mistakes of dominant students occurred at the level of basic algebraic process skills. A concrete example can be seen in the Data 7 document where students incorrectly describe the binomial square shape of the fraction denominator. They wrote the result of the squared by ignoring the existence of the middle quarter of the result of cross-multiplication of the two components of the algebra. Errors in this stage of process skills are also exacerbated by the inability of students to accurately distribute negative signs. As a result, the elimination phase of the same type in the numerator section results in an erroneous constant value and damages the final result. The weakness of students is not in the inability to read mathematical symbols but in the ability to process operations (Scherer et al., 2016).

In addition to differential material, this research area also evaluates the extent of students' mastery of the concept of integral calculus operations. In the case of indeterminate integral questions, student answer sheets generally reflect a uniform and systematic pattern of understanding. The students were able to apply the basic integration rules by raising the rank of the variable and dividing it by the value of the new coefficient. The transition of the way of thinking from derived operations to integral operations has been proven to be customizable without causing significant logical distortions (Bednar & Welch, 2020; Elahi et al., 2023; Shidaganti et al., 2023). The writing of integration constants in the form of capital letters at the end of mathematical expressions has also been applied in an orderly manner by students. Some students choose to leave the final result in the form of fractions that have not been perfectly simplified. However, this does not detract from the validity of the integration steps they have taken from the point of view of mathematical assessment. Positive achievements in this indefinite integral material become valuable cognitive capital before students step to the next level.

The real challenge of calculus begins to surface when students have to execute operations on integral problems, of course. (Bressoud et al., 2016; Nedaei et al., 2022). The characteristics of integral questions in this exam require the numerical conversion of the upper and lower limits into the form of fractions. The cognitive workload of students jumps very sharply because they are forced to equate the denominator of fractions until they touch the number of hundreds. Based on the visual recording of Data 2, the process of equalizing denominators to certain fractional values triggered the birth of many revision doodles. Students experience the participant discontinued procedural checking after several unsuccessful revision attempts in the middle of the road, which triggers the occurrence of careless basic arithmetic calculations. Failure to perform independent integer appointments such as the number seven is one of the factors that damages the accuracy of the final value of the fraction. The discrepancy in the results of numerical calculations between students proves that their main obstacle lies in the accuracy of the manual calculation of fractions. So, the biggest obstacle for students in this section is purely due to the factor of basic arithmetic proficiency and not because of the blindness of integral concepts.

The most critical and worrying findings of this study lie in the evaluation part of the theoretical understanding of differential equations. Students are asked independently to construct examples of ordinary and partial differential equations based on criteria of certain orders and degrees (Bleecker, 2018). The results of data analysis show that there is a very severe theoretical gap where seven of eight participants students experience complete failure. The subject of Djundjuma Tuahena in Data 10 actually wrote an ordinary algebraic function that is completely empty of differential symbols. He thought that a standard polynomial expression without a derivative symbol qualified as a form of ordinary differential equation. Meanwhile, other students who tried to include derivative symbols still the participant was unable to identify an appropriate alternative strategy to match the order level and the rank of the question's degree. They seemed confused to separate the concept of the highest derivative and the concept of outer rank attached to the differential symbol. This mass failure is clear evidence that being proficient in mechanical calculations does not guarantee that students understand the essence of the definition of lecture material.

The impact of this weak understanding of theoretical concepts will greatly affect students' success in more complex advanced lecture materials. Partial differential equations demand an understanding of multidimensional space that cannot be solved with the memorization of dead formulas alone (Becker, 2026; Brigo & Pistone, 2017). If students are unable to distinguish the basic structure of GDP and PDP from the start, they will have difficulty determining a valid method of settlement. The inability to identify orders and degrees will also make students wrong in applying the existence theorem and singularity of equation solutions. This condition requires a change in the teaching paradigm from the original outcome-oriented to conceptual process-oriented. The cultivation of the philosophy behind the writing of mathematical symbols must be emphasized repeatedly in every face-to-face lecture in the classroom (Engelbrecht et al., 2020; Jin, 2023). The practice questions given should not only focus on the mechanical aspect but must contain variations in the analysis of the form of equations. Through these improvement steps, fatal misconceptions about the basic elements of differential are expected to be minimized in the future.

As a final conclusion, this study succeeded in mapping a complete portrait of students' mathematical competence in taking differential equation courses. Mature instrumental skills on basic derivative material have proven to be unable to compensate for theoretical conceptual weaknesses and the rigor of fractional arithmetic. The researcher recommends the need to restructure the teaching module that is able to harmonize aspects of the calculation procedure with strengthening relational understanding in a balanced manner. Teachers are advised to insert refresher material on manual fraction operations and multiplication of binomial algebra before moving on to advanced calculus material. Providing periodic diagnostic tests at the beginning of each chapter is very important to detect the location of student misconceptions early. Students should also be encouraged to more actively explore the physical meaning of each differential notation they write on paper. The collaboration between strong theoretical understanding and high computational accuracy is the key to successful learning of advanced mathematics (Alam & Mohanty, 2024; Rach & Ufer, 2020). It is hoped that the results of this research can be a valuable reference for lecturers to improve the quality of calculus learning in the future.

CONCLUSION

This study identified the characteristics of retractive thinking demonstrated by mathematics education students when solving differential equation problems. The findings indicate that retractive thinking is reflected through three main characteristics: error recognition, procedural re-examination, and solution revision. Students showed varying levels of retractive thinking, ranging from simple correction of computational errors to more comprehensive revisions involving conceptual reconsideration of solution strategies. These findings suggest that retractive thinking functions as a metacognitive mechanism that helps students monitor, evaluate, and regulate their mathematical reasoning. The study also revealed a gap between students' procedural competence and their conceptual understanding of differential equation structures. Therefore, differential equation instruction should emphasize not only procedural skills but also reflective and self-regulatory learning processes. This study was limited to a purposive sample of eight mathematics education students from a single university and a single course context. Consequently, the findings provide analytical insights into retractive thinking rather than statistical generalizations and may serve as a foundation for future research and instrument development.

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A contributed to conceptualization, methodology, supervision, validation, writing – original draft, and writing – review & editing. HI & IKW contributed to methodology, formal analysis, investigation, resources, and writing – review & editing. A contributed to data curation, investigation, project administration, and writing – original draft. All authors have read and approved the final version of the manuscript.

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