



Islamic Reflective Realistic Mathematics Learning for Enhancing Students' Mathematical Representations: A Quasi-Experimental Study

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ARTICLE INFO	ABSTRACT
Article history: Submitted: July 02, 2026 Final Revised: August 29, 2026 Accepted: September 02, 2026 Published: September 12, 2026	Purpose: This study examined whether Islamic Reflective Realistic Mathematics Learning (IRRML), an instructional framework integrating reflective learning, Realistic Mathematics Education (RME), and Islamic values, was associated with stronger mathematical representation performance among Grade VII students.
Keywords: Islamic Reflective Realistic Mathematics Learning; mathematical representations; Realistic Mathematics Education; reflective learning; Islamic values.	Methods: A quantitative quasi-experimental pretest-posttest non-equivalent control group design was implemented with 64 students at an Islamic junior secondary school in Indonesia. Two intact classes were assigned to an experimental group receiving IRRML ($n = 32$) and a control group receiving conventional teacher-centered instruction ($n = 32$). Mathematical representations were assessed through an eight-item essay test covering visual, symbolic, verbal, and multiple-representation translation indicators. Data were analyzed using descriptive statistics and ANCOVA, with pretest scores treated as a covariate.
	Findings: The IRRML group obtained a higher posttest mean (84.72 ± 7.63) than the control group (71.46 ± 8.21). After adjustment for pretest scores, the learning model was significant, $F(1,61) = 36.84$, $p < .001$, partial $\eta^2 = .377$, and the reported standardized effect was large (Cohen's $d = 1.62$). Post-intervention scores favored the experimental group across all four representation indicators.
	Research Implications: Within this study context, IRRML provided a structured way to combine contextual problem situations, reflective dialogue, multiple-representation construction, and value-oriented learning. The findings support further investigation of this integrated approach across schools, mathematical topics, and educational settings.
	Originality: The study operationalizes reflective learning, RME, and Islamic values within one instructional sequence and evaluates the framework in relation to students' mathematical representations.
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INTRODUCTION

Mathematics education increasingly asks students to do more than execute procedures: learners are expected to interpret situations, construct mathematical objects, and communicate relationships through appropriate representations. Representational competence is central to this work because the same mathematical idea may need to be expressed visually, symbolically, verbally, or contextually depending on the problem. International evidence nevertheless shows that students often experience difficulty coordinating these forms, particularly when a task requires movement from a real situation to a diagram, expression, or written explanation (Al Farra et al., 2022; Ayeh, 2025; Cotič et al., 2024; Rolfes et al., 2022). Studies of multiple-representation environments similarly indicate that providing several representations does not automatically produce understanding; students must recognize and build meaningful connections among them (Bronkhorst et al., 2021; Cordero-Siy & Ghousseini, 2022; Friesen & Kuntze, 2021; Haj-Yahya & Haj-Yahya, 2026; Tan et al., 2023). For junior secondary learners, this challenge is pedagogically important

because representational flexibility supports later reasoning, modelling, communication, and problem solving. The issue is therefore not simply whether students can produce a correct answer, but whether they can explain and transform the mathematical meaning that underlies it.

In classroom practice, representational difficulties are often reinforced when instruction gives priority to formulas and worked procedures before students have opportunities to construct meaning from situations. Students may obtain numerical answers while remaining uncertain about why a diagram, symbol, table, or verbal statement represents the same mathematical relationship (Manoharan & Kaur, 2023; Reyna & Brainerd, 2023). Research on representation-rich instruction suggests that learning improves when students are required to compare forms, explain their choices, and justify transitions between representations rather than merely view several forms side by side (Kokkonen & Schalk, 2021; Rolfes et al., 2022; Tan et al., 2023). This point is especially relevant for geometry topics such as quadrilaterals, in which learners must coordinate visual properties, verbal definitions, symbolic relationships, and contextual models. The instructional problem is therefore one of coordination and meaning-making, not simply exposure to additional representations. A learning design intended to improve mathematical representations needs to create repeated opportunities for students to construct, test, revise, and translate their own representations.

Reflective learning offers one route for addressing this problem because it directs students' attention to the reasoning used during problem solving. When students revisit a solution, compare alternative strategies, and evaluate whether a representation is consistent with the problem, reflection can make otherwise implicit reasoning available for examination (Kholid et al., 2022; Tee et al., 2021). Metacognitive approaches in mathematics have also been associated with stronger critical and reflective thinking when learners are prompted to plan, monitor, and evaluate their work (Santosa et al., 2026; Syaiful et al., 2022; Tachie, 2019). These processes are relevant to representation because translating from one form to another requires students to monitor whether mathematical meaning has been preserved. Reflection alone, however, does not specify the kinds of situations from which mathematical ideas should be developed. For that reason, a reflective approach may be strengthened when it is embedded in contextual activity that gives students something meaningful to mathematize and reconsider.

Realistic Mathematics Education (RME) provides such a context by treating meaningful situations as starting points for progressive mathematization rather than introducing formal procedures as finished knowledge. Empirical studies have associated RME with gains in conceptual understanding, mathematical communication, reasoning, self-efficacy, and problem-solving performance across different educational settings (Caraan et al., 2023; Palinussa et al., 2021; Purnamatati et al., 2023; Rangkuti et al., 2024; Ventistas et al., 2025; Yohannes & Chen, 2024). Recent work also shows that realistic or context-based approaches can support the movement from informal models toward more formal mathematical representations, although the size and form of the benefit depend on task design and learner characteristics (Hartmann et al., 2021; Holenstein et al., 2022; Özer Şanal & Elmalı, 2024; Ventistas et al., 2025). Within RME, representations are not treated only as products of completed reasoning; they function as tools through which students organize a context, communicate relationships, and progressively formalize ideas. This feature makes RME compatible with reflective learning because students can examine how an initial contextual model changes as their reasoning becomes more formal. The combination is therefore theoretically relevant to representational competence: realistic situations generate material for mathematical construction, while reflection can help students evaluate and coordinate the representations produced during that construction.

The educational context of the present study adds a further dimension. In Islamic schools, mathematics learning takes place within an institutional environment in which cognitive development and value formation are commonly expected to coexist (Nasucha et al., 2023). Studies on Islamic value integration argue that values become more educationally meaningful when they are connected to classroom practices and decision making rather than added as detached quotations or illustrations (Asnawi et al., 2026; Mulya & Faradiba, 2025; Setiawan et al., 2025; Ulpah et al., 2025). More broadly, culturally responsive mathematics research has shown that learners' cultural experiences can become resources for mathematical activity when instruction maintains disciplinary rigor while making contexts meaningful to students (Ngoako & Machaba, 2026; Register et al., 2022; Shultz et al., 2024). In the present framework, Islamic values are therefore positioned as part of the learning environment through expectations such as honesty in reasoning, responsibility for group work, respect for alternative solutions, and reflection on order and balance. This design does not assume that religious content independently causes higher mathematical achievement; rather, it examines an integrated instructional setting in which value-oriented participation accompanies contextual and reflective mathematical work.

Previous research provides support for each of these components, but the literature remains fragmented in ways that matter for mathematical representation. Reflective and metacognitive studies have mainly emphasized reasoning,

problem solving, self-regulation, or critical thinking (Guo et al., 2024; Kholid et al., 2022; Santosa et al., 2026; Syaiful et al., 2022; Tee et al., 2021), whereas RME research has more often examined achievement, conceptual understanding, problem solving, communication, or self-efficacy (Caraan et al., 2023; Palinussa et al., 2021; Rangkuti et al., 2024; Ventistas et al., 2025; Yohannes & Chen, 2024). Research on multiple representations, meanwhile, demonstrates the importance of coordinating representational forms but does not typically combine this objective with reflective learning and an explicitly value-oriented school context (Bronkhorst et al., 2021; Cordero-Siy & Ghousseini, 2022; Rolfes et al., 2022; Setianingsih et al., 2026; Tan et al., 2023). Studies of Islamic value integration in mathematics have largely focused on curriculum, conceptual integration, or broader educational purposes rather than testing a combined instructional sequence against a comparison group. The resulting gap is not the absence of evidence for reflection, RME, representation, or value integration individually; it is the limited evidence on how these elements operate together within one classroom model focused specifically on representational competence. Addressing that gap requires an instructional framework whose components are explicit enough to be implemented and evaluated rather than discussed only at a conceptual level.

Accordingly, this study examined the effectiveness of Islamic Reflective Realistic Mathematics Learning (IRRML) in enhancing Grade VII students' mathematical representations in an Islamic junior secondary school. IRRML integrates Islamic contextual orientation, realistic mathematical exploration, reflective dialogue, construction of multiple representations, and reflection with value internalization within a five-phase instructional sequence. The study compared students receiving IRRML with students receiving conventional teacher-centered instruction using a quasi-experimental pretest-posttest non-equivalent control group design. The primary question was whether posttest mathematical representation scores differed between groups after controlling for pretest performance. Theoretically, the study tests whether contextual mathematization and reflective regulation can be organized around explicit representational work within a culturally meaningful learning environment. Practically, it provides a classroom procedure that mathematics teachers can evaluate and adapt while preserving the distinction between the observed effect of the integrated IRRML package and the untested independent effects of its individual components.

METHOD

Research Design, Setting, and Participants

This study used a quantitative quasi-experimental pretest-posttest non-equivalent control group design to examine differences in mathematical representation performance between students who received IRRML and those who received conventional instruction. The design was selected because the school maintained intact classes and individual random assignment was not feasible. The study was conducted at SMP Muhammadiyah 3 Bandar Lampung, Lampung, Indonesia, during the first semester of the 2026/2027 academic year over eight instructional weeks [PERLU VERIFIKASI: this period is inconsistent with the 2025 article-history dates shown on the first page]. The population comprised all Grade VII students enrolled at the school. Two intact classes were selected through cluster random sampling after prior-semester mathematics achievement had been checked for comparability, producing a final sample of 64 students: 32 in the experimental group and 32 in the control group. Students who did not complete either the pretest or posttest or attended fewer than 80% of the instructional meetings were excluded from the final analysis.

IRRML Intervention

The experimental treatment was an integrated instructional framework named Islamic Reflective Realistic Mathematics Learning (IRRML), which combined reflective learning, Realistic Mathematics Education (RME), and Islamic values within a five-phase sequence. Islamic values were embedded in classroom activity through expectations related to honesty, responsibility, collaboration, respect for alternative reasoning, and reflection on order and harmony rather than being added as separate religious content. The phases were Islamic Contextual Orientation, Realistic Mathematical Exploration, Reflective Mathematical Dialogue, Multiple Representation Construction, and Reflection and Value Internalization. Table 1 presents the learning activities and intended outcomes for each phase. Because the intervention combined several pedagogical components simultaneously, subsequent interpretation concerns the IRRML package as a whole rather than the independent effect of any single component.

Table 1. Instructional Syntax of Islamic Reflective Realistic Mathematics Learning (IRRML)

Phase	Learning Activities	Expected Learning Outcomes
1. Islamic Contextual Orientation	Students are introduced to contextual problems connected with everyday life and Islamic values relevant to the lesson.	Students develop learning motivation and recognize mathematics within meaningful contexts.

2. Realistic Mathematical Exploration	Students investigate contextual situations, identify mathematical relationships, and formulate initial mathematical models collaboratively.	Students transform contextual situations into mathematical representations.
3. Reflective Mathematical Dialogue	Students discuss alternative solution strategies, justify mathematical reasoning, and evaluate peer responses through guided reflection.	Students develop reflective thinking and strengthen conceptual understanding.
4. Multiple Representation Construction	Students construct and translate mathematical ideas among verbal, visual, symbolic, graphical, and contextual representations.	Students demonstrate flexibility in producing multiple mathematical representations.
5. Reflection and Value Internalization	Students evaluate their learning processes, summarize conceptual understanding, and reflect upon Islamic values demonstrated during learning.	Students consolidate conceptual understanding while internalizing positive learning values.

Instructional Control Condition

Students in the control group received conventional teacher-centered mathematics instruction consisting of teacher explanation, worked examples, guided exercises, and individual assignments. The mathematical content, learning objectives, instructional duration, and assessment procedures were held the same across groups as reported in the study. The instructional approach was therefore the planned between-group difference. The manuscript does not provide additional information about whether the same teacher taught both classes or about possible teacher effects; consequently, this factor should be considered when interpreting the findings.

Research Instruments

Data were collected using the Mathematical Representations Test, classroom observation sheets, learning implementation checklists, and instructional documentation. The primary outcome measure was an eight-item open-ended Mathematical Representations Test developed with reference to the NCTM Representation Standard and the mathematics education literature. The test assessed four indicators: visual representation, symbolic representation, verbal representation, and multiple-representation translation. Visual items required students to express mathematical ideas through figures, diagrams, graphs, or geometric illustrations; symbolic items required mathematical expressions, equations, or algebraic models; verbal items required written explanations of reasoning and solution processes; and translation items required movement among verbal, visual, symbolic, and contextual forms. Table 2 summarizes the operational definition of each indicator. Because the instrument used essay responses, a scoring rubric and information about scorer reliability would strengthen replicability; these details were not available in the source manuscript and were therefore not added.

Table 2. Indicators of Mathematical Representations

Indicator	Operational Description
Visual Representation	Expressing mathematical ideas through figures, diagrams, graphs, or geometric illustrations.
Symbolic Representation	Translating contextual situations into mathematical expressions, equations, or algebraic models.
Verbal Representation	Explaining mathematical reasoning and solution processes using written language.
Multiple Representation Translation	Transforming mathematical information accurately among verbal, visual, symbolic, and contextual representations.

Instrument Validity and Reliability

Content validity was evaluated through expert judgment involving three specialists in mathematics education, curriculum development, and educational assessment. The experts considered item relevance, clarity, representativeness, and alignment with the intended learning outcomes, and the items were revised following their review. The source manuscript reports that construct validity was examined using Confirmatory Factor Analysis and that all retained indicators had standardized factor loadings above 0.50. Internal consistency was reported as Cronbach's $\alpha = 0.91$. A pilot study was also conducted with students from another school described as having similar academic characteristics to examine item difficulty, discrimination indices, and administration procedures. The available manuscript does not report the pilot sample size, individual factor loadings, model-fit indices, or item-level difficulty and discrimination values, so these statistics were not reconstructed or inferred.

Data Collection Procedure

Data collection began with administrative preparation, teacher orientation, instrument validation, and classroom scheduling. At the first meeting, both groups completed the Mathematical Representations Test as a pretest under the same reported testing conditions. The experimental class then completed eight instructional meetings using the IRRML sequence, while the control class received conventional instruction over the same period. Two independent observers used structured implementation checklists during each experimental session; the manuscript reports inter-observer agreement above 90%. At the end of the intervention, both groups completed the same instrument as a posttest. Student responses, observation records, instructional documentation, and attendance records were then compiled and checked before analysis.

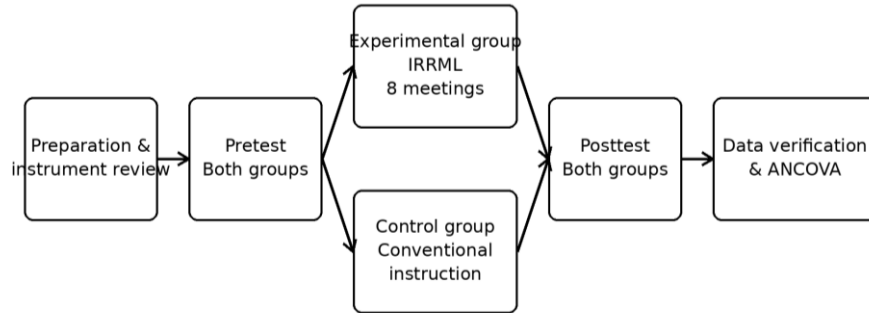


Figure 1. Study procedure and analytical sequence

Data Analysis

Statistical analysis was conducted using IBM SPSS Statistics version 29. Descriptive statistics were used to summarize pretest and posttest performance. The Shapiro-Wilk test was used to assess normality and Levene's test to assess homogeneity of variance. The primary group comparison used Analysis of Covariance (ANCOVA), with posttest mathematical representation scores as the dependent variable, instructional group as the factor, and pretest scores as the covariate. Statistical significance was evaluated at $\alpha = .05$, and effect magnitude was reported using partial eta squared and Cohen's d. The source manuscript did not report a test of homogeneity of regression slopes or adjusted means with confidence intervals; these values were not estimated because the underlying student-level dataset was not available.

RESULTS

4.1 Descriptive Statistics of Students' Mathematical Representations

Table 3 summarizes students' performance before and after the intervention. The experimental and control groups began with similar mean pretest scores (56.84 and 57.31, respectively). At posttest, the experimental group obtained a mean of 84.72 (SD = 7.63), compared with 71.46 (SD = 8.21) in the control group. The corresponding raw gain scores were 27.88 for the experimental group and 14.15 for the control group. These descriptive differences indicate a larger observed increase in the IRRML group, but the inferential comparison was based on ANCOVA so that baseline performance could be taken into account. The descriptive results therefore provide context for, rather than replace, the adjusted group comparison reported below.

Table 3. Descriptive Statistics of Mathematical Representations

Group	N	Pretest Mean $\pm$ SD	Posttest Mean $\pm$ SD	Gain Score
Experimental (IRRML)	32	56.84 $\pm$ 9.18	84.72 $\pm$ 7.63	27.88
Control	32	57.31 $\pm$ 8.94	71.46 $\pm$ 8.21	14.15

4.2 Assumption Testing

Before the primary hypothesis test, normality and homogeneity of variance were examined using the procedures reported in the source dataset. The Shapiro-Wilk results for the experimental and control groups were non-significant ($p = .421$ and $p = .318$, respectively), and Levene's test was also non-significant, $F = 0.81$, $p = .372$. These results were consistent with normality and equal-variance assumptions for the reported scores. Table 4 presents the assumption-test

statistics available in the manuscript. A test of homogeneity of regression slopes, which is also relevant to ANCOVA, was not reported in the available data and therefore was not reconstructed.

Table 4. Assumption Testing

Test	Statistic	p-value	Decision
Shapiro–Wilk (Experimental)	0.972	.421	Normal
Shapiro–Wilk (Control)	0.968	.318	Normal
Levene's Test	0.81	.372	Homogeneous

4.3 Effect of IRRML on Students' Mathematical Representations

ANCOVA was used to compare posttest mathematical representation scores while controlling for pretest performance. The learning model effect was statistically significant, $F(1,61) = 36.84$, $p < .001$, partial $\eta^2 = .377$, after adjustment for the covariate. The pretest covariate was also significant, $F = 12.47$, $p = .001$, partial $\eta^2 = .170$. The reported Cohen's d for the between-group difference was 1.62, indicating a large standardized effect. Taken together, the available statistics show that students in the IRRML group had substantially higher post-intervention performance than students in the conventional group after controlling for baseline performance. Because adjusted means and confidence intervals were not provided in the source manuscript, the magnitude of the difference is reported using the available F statistic and effect-size estimates rather than reconstructed values.

Table 5. ANCOVA Results

Source	F	p	Partial η^2
Pretest	12.47	.001	.170
Learning Model	36.84	< .001	.377

4.4 Post-Intervention Performance across Representation Indicators

Post-intervention scores favored the experimental group on each representation indicator. The experimental and control means were 85.63 and 72.48 for visual representation, 87.41 and 74.96 for symbolic representation, 81.54 and 70.33 for verbal representation, and 84.29 and 68.12 for multiple-representation translation. The largest posttest mean difference occurred for multiple-representation translation, followed by visual representation, symbolic representation, and verbal representation. These values describe the pattern of post-intervention performance but should not be interpreted as indicator-specific gains because pretest scores for each indicator were not reported. Accordingly, the results support the conclusion that the experimental group performed better across representational forms at posttest, while claims about which indicator improved most require indicator-level pretest data.

Table 6. Mean Scores by Representation Indicator

Indicator	Experimental	Control
Visual Representation	85.63	72.48
Symbolic Representation	87.41	74.96
Verbal Representation	81.54	70.33
Multiple Representation Translation	84.29	68.12

4.5 Implementation Fidelity

Observation records indicated that all five IRRML phases were implemented with high reported fidelity. Phase-level fidelity ranged from 93.8% for Reflective Mathematical Dialogue to 95.6% for Multiple Representation Construction, with an overall value of 94.8%. Inter-observer agreement exceeded 90% across the instructional sessions. These observations support the interpretation that the planned IRRML sequence was implemented consistently in the experimental class. They do not, however, exclude other classroom influences such as teacher enthusiasm, student group dynamics, or novelty effects. Table 7 presents the reported fidelity values for each instructional phase.

Table 7. Implementation Fidelity

Phase	Fidelity (%)
Islamic Contextual Orientation	95.2
Realistic Mathematical Exploration	94.7
Reflective Mathematical Dialogue	93.8
Multiple Representation Construction	95.6
Reflection and Value Internalization	94.9
Overall	94.8

DISCUSSION

The central finding is that students who received IRRML achieved higher mathematical representation scores than students who received conventional instruction after pretest performance was controlled. The size of the reported group effect, partial $\eta^2 = .377$ with Cohen's $d = 1.62$, indicates that the difference was educationally substantial within this sample. The result is consistent with research showing that students benefit when mathematical ideas are developed through contexts and representations that require active meaning-making rather than immediate reliance on formal procedures (Rolfes et al., 2022; Ventistas et al., 2025). It also aligns with evidence that realistic and problem-oriented approaches can support conceptual development when students are asked to mathematize meaningful situations (Caraan et al., 2023; Palinussa et al., 2021; Rangkuti et al., 2024; Torres-Peña et al., 2025; Yohannes & Chen, 2024). In IRRML, contextual exploration was followed by reflective dialogue and explicit construction of multiple representations, so students repeatedly moved between informal understanding and more formal mathematical expression. The findings therefore suggest that the advantage of the intervention is best interpreted as an effect of this coordinated instructional sequence rather than as evidence for any one component in isolation.

The pattern across representation indicators helps clarify what that coordination may have looked like. At posttest, the experimental group outperformed the control group in visual, symbolic, verbal, and multiple-representation translation scores, with the largest between-group difference appearing in translation across representations. Research on multiple representations has repeatedly shown that students need more than exposure to diagrams, symbols, tables, or words; they must build connections that preserve mathematical meaning across forms (Cordero-Siy & Ghouseini, 2022; Haj-Yahya & Haj-Yahya, 2026; Kokkonen & Schalk, 2021; Rolfes et al., 2022; Setianingsih et al., 2026; Tan et al., 2023). The IRRML sequence made such movement an explicit classroom activity through contextual exploration, dialogue, and representation construction. This mechanism is plausible because students first interpreted a situation, then compared reasoning with peers, and finally reconstructed the idea in several forms. However, the indicator table contains posttest means only, so the present data do not establish which representational skill improved the most from baseline. What can be concluded is that the experimental group finished the intervention with stronger performance across all four measured forms.

Reflective learning provides one explanation for why the intervention may have supported representational coordination. Moving from a diagram to an algebraic relation or from a contextual statement to a symbolic model requires monitoring whether essential relationships have been preserved, and that monitoring is inherently metacognitive. Kholid et al. (2022) showed that reflective thinking in mathematical problem solving involves changes in how students examine and reorganize their reasoning, while Tee et al. (2021) linked self-regulation to mathematics achievement through monitoring and control processes. Experimental and synthesis research on reflective and metacognitive learning has likewise reported benefits for higher-order mathematical thinking when students are prompted to plan, evaluate, and revise their reasoning (Santosa et al., 2026; Syaiful et al., 2022). Within IRRML, Reflective Mathematical Dialogue required students to justify strategies and evaluate peer responses before completing multiple-representation construction. This sequencing may have reduced the tendency to treat representations as separate answer formats by making the relationships among them a subject of discussion. Because the study did not directly measure metacognition, this explanation remains a theoretically grounded interpretation rather than a tested mediating mechanism.

RME contributes a complementary mechanism by giving students contextual material from which mathematical representations can be developed. Instead of beginning with a finished symbolic procedure, realistic problems allow learners to organize situations, identify relationships, and progressively formalize the mathematics involved. This interpretation is consistent with recent studies reporting improvements in problem solving, conceptual understanding, communication, and related mathematical outcomes through realistic or context-based tasks (Caraan et al., 2023; Hartmann et al., 2021; Purnamatati et al., 2023; Rangkuti et al., 2024; Ventistas et al., 2025; Yohannes & Chen, 2024), as well as work showing that learner characteristics such as self-efficacy can matter in mathematical modelling (Holenstein et al., 2022). The present study extends that line of work by making representational translation an explicit outcome within the instructional sequence. In practical terms, the contextual task was not treated as decoration; it served as the starting point for visual, verbal, and symbolic modelling. The result supports the view that contextualization is most useful for representation when students are also required to compare, refine, and justify the models they produce.

The value-oriented dimension of IRRML should be interpreted more cautiously. The intervention embedded Islamic values through classroom expectations related to honesty, responsibility, cooperation, respect for alternative reasoning, and reflection on order and harmony. Research on Islamic value integration has argued that values are more meaningful

when they shape learning activity rather than appearing only as quotations or supplementary messages (Asnawi et al., 2026; Mulya & Faradiba, 2025; Setiawan et al., 2025; Ulpah et al., 2025). This orientation also resonates with culturally responsive mathematics research, which emphasizes the use of learners' cultural contexts as resources for participation without reducing mathematical rigor (Ngoako & Machaba, 2026; Register et al., 2022; Shultz et al., 2024). In the present school, such value framing may have supported participation and reflection because it was familiar to the institutional context. Yet the design included no RME-plus-reflection condition without Islamic values, so the study cannot isolate an independent causal contribution of value integration. The appropriate conclusion is that value-oriented practice formed part of the effective IRRML package and may have supported the learning environment in which representational work occurred.

The study also adds to representation research by connecting representational fluency with a sequence of contextualization and reflection rather than treating multiple representations as an isolated instructional technique. Prior work shows that multi-representational learning can improve understanding, but its effects depend on the representation types, learning content, and opportunities students have to coordinate them (Bronkhorst et al., 2021; Kokkonen & Schalk, 2021; Rolfes et al., 2022). Classroom research likewise suggests that teachers need to make relationships among representations visible and discussable if students are to use them as connected mathematical resources (Cordero-Siy & Ghousseini, 2022; Friesen & Kuntze, 2021; Tan et al., 2023). IRRML addresses this issue by assigning a distinct phase to Multiple Representation Construction after contextual exploration and reflective dialogue. Conceptually, the model therefore positions representation as both a means of developing understanding and an outcome through which that understanding becomes observable. This organization extends the literature by showing one way to integrate representation work with RME and reflection in a value-oriented junior secondary setting. The contribution is instructional and conceptual rather than evidence that IRRML constitutes a universally superior model. Several contextual factors may also help explain the magnitude of the observed effect. The intervention occurred in one Islamic junior secondary school, and students were already situated in an environment where value-based learning may have been familiar. High implementation fidelity indicates that the planned phases were delivered consistently, but fidelity does not eliminate teacher enthusiasm, novelty effects, peer dynamics, or differences in how the two intact classes responded to instruction; classroom participation can also be shaped by how opportunities for student talk are distributed during discussion (Zamir & Asterhan, 2026). The study also focused on quadrilaterals, a topic that naturally invites visual and symbolic coordination, so the same effect size should not be assumed for topics with different representational demands. In addition, the source manuscript does not report whether the same teacher taught both groups, the test of homogeneity of regression slopes, indicator-level pretest scores, or detailed scorer reliability for the essay instrument. These limitations narrow the strength of causal and generalizable claims. The results are therefore strongest as evidence that the implemented IRRML package was associated with better post-intervention representation performance in the participating classes under the reported study conditions.

For practice, the findings suggest that teachers who wish to strengthen mathematical representations may benefit from organizing lessons around a sequence in which students first interpret meaningful situations, then explain and challenge reasoning, construct several representational forms, and finally reflect on both mathematical and learning processes. Such a sequence differs from simply presenting several representations or adding contextual examples after formal instruction. For research, the next step is to test IRRML across multiple schools, instructors, mathematical domains, and student populations, ideally with designs that can separate the contributions of reflection, RME, and value integration. Future studies should also report indicator-level pretest and posttest results, adjusted ANCOVA means with confidence intervals, complete assumption checks, and inter-rater reliability for open-ended scoring. Qualitative evidence from classroom discourse or student work could further examine how students actually move among representations during the reflective phases. These extensions would clarify whether the mechanisms proposed here are observable in students' reasoning and whether the framework transfers beyond the specific institutional and mathematical context of this study.

CONCLUSION

This quasi-experimental study found that Grade VII students who received Islamic Reflective Realistic Mathematics Learning (IRRML) achieved higher mathematical representation performance than students who received conventional instruction after pretest scores were controlled. The experimental group also obtained higher post-intervention means across visual, symbolic, verbal, and multiple-representation translation indicators. Within the limits of the design, these results support IRRML as an integrated instructional approach in which realistic contexts, reflective dialogue, explicit representation construction, and value-oriented classroom practices are organized within one learning sequence. The

findings should not be used to infer the independent effect of Islamic values, reflective learning, or RME because those components were not experimentally separated. Generalization is also limited by the single-school setting, the sample of 64 students, the use of intact classes, and the focus on quadrilaterals. Further research across schools, teachers, and mathematical topics, with fuller reporting of ANCOVA assumptions and open-ended scoring reliability, is needed to determine the stability and transferability of the observed effect. For classroom practice, the study provides a structured model that can be evaluated and adapted when the instructional goal is to help students construct, explain, and translate mathematical ideas across multiple forms of representation.

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REFERENCE

- Al Farra, N. K., Al Owais, N. S., & Belbase, S. (2022). Computational, Logical, Argumentative, and Representational Thinking in the United Arab Emirates Schools: Fifth Grade Students' Skills in Mathematical Problem Solving. *Mathematics Teaching Research Journal*, 14(1), 215–252.
- Asnawi, A. R., Supandi, M. D., & Farabi, F. A. A. (2026). The Qur'an and Mathematics: The Dimension of Tawhid and Knowledge in the Perspective of Afzalur Rahman. *QiST: Journal of Quran and Tafseer Studies*, 5(2), 477–496. <https://doi.org/10.23917/qist.v5i2.13865>
- Ayeh, I. G. (2025). Students' Mathematics Conceptual Challenges: Exploring Students' Thinking, Understanding, and Misconceptions in Functions and Graphs. *European Journal of Science and Mathematics Education*, 13(3), 191–206. <https://doi.org/10.30935/scimath/16596>
- Bronkhorst, H., Roorda, G., Suhre, C., & Goedhart, M. (2021). Student development in logical reasoning: Results of an intervention guiding students through different modes of visual and formal representation. *Canadian Journal of Science, Mathematics and Technology Education*, 21, 378–399. <https://doi.org/10.1007/s42330-021-00148-4>
- Caraan, D. R., Dinglasan, J. K., & Ching, D. (2023). Effectiveness of Realistic Mathematics Education Approach on Problem-Solving Skills of Students. *International Journal of Educational Management and Development Studies*, 4(2), 64–87. <https://doi.org/10.53378/352980>
- Cordero-Siy, E., & Ghouseini, H. (2022). Supporting understanding using representations. *Mathematics Teacher: Learning and Teaching PK–12*, 115(6), 394–403.
- Cotič, M., Doz, D., Jenko, M., & Žakelj, A. (2024). Mathematics education: What was it, what is it, and what will it be? *International Electronic Journal of Mathematics Education*, 19(3), em0783. <https://doi.org/10.29333/iejme/14663>
- Friesen, M. E., & Kuntze, S. (2021). How context specific is teachers' analysis of how representations are dealt with in classroom situations? Approaching a context-aware measure for teacher noticing. *ZDM–Mathematics Education*, 53, 181–193. <https://doi.org/10.1007/s11858-020-01204-3>
- Guo, R., Jantharajit, N., & Thongpanit, P. (2024). Construct an Instructional Approach Based on Collaborative Learning and Reflective Learning for Enhance Students' Analytical Thinking and Critical Thinking Skills. *Asian Journal of Contemporary Education*, 8(2), 115–125.
- Haj-Yahya, A., & Haj-Yahya, H. A. (2026). Exploring the effects of GeoGebra on developing students' mathematical concept definitions and images. *International Journal of Mathematical Education in Science and Technology*, 1–29. <https://doi.org/10.1080/0020739X.2026.2648022>
- Hartmann, L.-M., Krawitz, J., & Schukajlow, S. (2021). Create your own problem! When given descriptions of real-world situations, do students pose and solve modelling problems? *ZDM–Mathematics Education*, 53, 919–935. <https://doi.org/10.1007/s11858-021-01224-7>
- Holenstein, M., Bruckmaier, G., & Grob, A. (2022). How do self-efficacy and self-concept impact mathematical achievement? The case of mathematical modelling. *British Journal of Educational Psychology*, 92(1), e12443. <https://doi.org/10.1111/bjep.12443>
- Kholid, M. N., Sa'Dijah, C., Hidayanto, E., & Permadi, H. (2022). Students' reflective thinking pattern changes and characteristics of problem solving. *Reflective Practice*, 23(3), 319–341. <https://doi.org/10.1080/14623943.2021.2025353>

- Kokkonen, T., & Schalk, L. (2021). One instructional sequence fits all? A conceptual analysis of the applicability of concreteness fading in mathematics, physics, chemistry, and biology education. *Educational Psychology Review*, 33, 797–821. <https://doi.org/10.1007/s10648-020-09581-7>
- Manoharan, M., & Kaur, B. (2023). Mathematics Teachers' Perceptions of Diagrams. *International Journal of Science and Mathematics Education*, 21(4), 1315–1337. <https://doi.org/10.1007/s10763-022-10312-3>
- Mulya, A., & Faradiba, S. (2025). Integration of Islamic Values in Mathematics Learning. *Jurnal Pendidikan Sains*, 13(4), 264–276. <https://doi.org/10.17977/jps.v13i42025p264>
- Nasucha, M. R., Khozin, K., & Thoifah, I. (2023). Synergizing Islamic Religious Education and Scientific Learning in the 21st Century: A Systematic Review of Literature. *Jurnal Pendidikan Agama Islam (Journal of Islamic Education Studies)*, 11(1), 109–130. <https://doi.org/10.15642/jpai.2023.11.1.109-130>
- Ngoako, R. N., & Machaba, F. M. (2026). Enacting culturally relevant pedagogy in a grade 10 mathematics classroom: An analysis of lessons on functions. *Eurasia Journal of Mathematics, Science and Technology Education*, 22(5), em2830. <https://doi.org/10.29333/ejmste/18507>
- Özer Şanal, S., & Elmalı, F. (2024). Effectiveness of realistic math education on mathematical problem-solving skills of students with learning disability. *European Journal of Special Needs Education*, 39(1), 109–126. <https://doi.org/10.1080/08856257.2023.2191110>
- Palinussa, A. L., Molle, J. S., & Gaspersz, M. (2021). Realistic Mathematics Education: Mathematical Reasoning and Communication Skills in Rural Contexts. *International Journal of Evaluation and Research in Education*, 10(2), 522–534. <https://doi.org/10.11591/ijere.v10i2.20640>
- Purnamatati, A. M. M., Usman, H., & Yunianingsih, E. (2023). Influence a Realistic Mathematics Education Approach and Motivation on Students' Mathematical Reasoning Ability. *Journal of Instructional Mathematics*, 4(1), 13–20. <https://doi.org/10.37640/jim.v4i1.1618>
- Rangkuti, A. N., Nasution, M., Amir, A., Dalimunthe, I. S., Samsuddin, S., Amin, M., Siregar, D., & Tarigan, M. R. M. (2024). The Effect of Realistic Mathematics Education Approach on Students' Conceptual Understanding. *Ta'dib*, 27(2), 287–298. <https://doi.org/10.31958/jt.v27i2.12933>
- Register, J., Fernandes, A., & Pugalee, D. (2022). Supporting preservice mathematics teachers' culturally responsive teaching: A focus on teaching for social justice. *Mathematics*, 10(6), 896. <https://doi.org/10.3390/math10060896>
- Reyna, V. F., & Brainerd, C. J. (2023). Numeracy, gist, literal thinking and the value of nothing in decision making. *Nature Reviews Psychology*, 2(7), 421–439. <https://doi.org/10.1038/s44159-023-00188-7>
- Rolfes, T., Roth, J., & Schnotz, W. (2022). Mono- and multi-representational learning of the covariational aspect of functional thinking. *Journal for STEM Education Research*, 5, 1–27. <https://doi.org/10.1007/s41979-021-00060-4>
- Santosa, Y. T., Mahmudah, M. H., Yulindra, D., & Pakpahan, A. W. (2026). Reflective Thinking and Self-Efficacy: A Meta-Analysis and Its Implications for Mathematics Learning. *International Journal of Review in Mathematics Education*, 52–77. <https://doi.org/10.23917/ijrime.14592>
- Setianingsih, R., Prayogo, M. R. A. Y., & Putri, A. M. Y. (2026). Multiple Mathematical Representations in Ethnomathematics-Based Realistic Mathematics Education: A Systematic Literature Review. *International Journal of Review in Mathematics Education*, 109–121. <https://doi.org/10.23917/ijrime.16307>
- Setiawan, S., Walidin, W., & Siregar, Z. A. B. (2025). Revitalization of Islamic Values-Based Curriculum: Integration of Religious Education in Mathematics Learning. *Electronic Journal of Education, Social Economics and Technology*, 6(2), 944. <https://doi.org/10.33122/ejeset.v6i2.944>
- Shultz, M., Close, E., Nissen, J., & Van Dusen, B. (2024). Enacting culturally relevant pedagogy when 'mathematics has no color': Epistemological contradictions. *International Journal of Research in Undergraduate Mathematics Education*, 10, 486–515. <https://doi.org/10.1007/s40753-023-00219-x>
- Syaiful, Huda, N., Mukminin, A., & Habibi, A. (2022). Using a metacognitive learning approach to enhance students' critical thinking skills through mathematics education. *SN Social Sciences*, 2, 31. <https://doi.org/10.1007/s43545-022-00325-8>
- Tachie, S. A. (2019). Meta-cognitive Skills and Strategies Application: How this Helps Learners in Mathematics Problem-solving. *Eurasia Journal of Mathematics, Science and Technology Education*, 15(5), em1702. <https://doi.org/10.29333/ejmste/105364>
-

- Tan, S., Clivaz, S., & Sakamoto, M. (2023). Presenting multiple representations at the chalkboard: Bansho analysis of a Japanese mathematics classroom. *Journal of Education for Teaching*, 49(4), 630–647. <https://doi.org/10.1080/02607476.2022.2150538>
- Tee, K. N., Leong, K. E., & Abdul Rahim, S. S. (2021). A Self-Regulation Model of Mathematics Achievement for Eleventh-Grade Students. *International Journal of Science and Mathematics Education*, 19(3), 619–637. <https://doi.org/10.1007/s10763-020-10076-8>
- Torres-Peña, R. C., Peña-González, D., Lara-Orozco, J. L., Ariza, E. A., & Vergara, D. (2025). Enhancing Numerical Thinking Through Problem Solving: A Teaching Experience for Third-Grade Mathematics. *Education Sciences*, 15(6), 667. <https://doi.org/10.3390/educsci15060667>
- Ulpah, M., Setiani, R. E., & Ramadani, N. (2025). The Integrating Islamic Values in The Mathematics Education Curriculum at UIN Saizu Purwokerto. *Circle: Jurnal Pendidikan Matematika*, 5(2), 218–231. <https://doi.org/10.28918/circle.v5i2.11738>
- Ventistas, G., Ventista, O. M., & Tsani, P. (2025). The impact of realistic mathematics education on secondary school students' problem-solving skills: A comparative evaluation study. *Research in Mathematics Education*, 27(3), 437–461. <https://doi.org/10.1080/14794802.2024.2306633>
- Yohannes, A., & Chen, H.-L. (2024). The effect of flipped realistic mathematics education on students' achievement, mathematics self-efficacy and critical thinking tendency. *Education and Information Technologies*, 29(13), 16177–16203. <https://doi.org/10.1007/s10639-024-12502-8>
- Zamir, A. C., & Asterhan, C. S. C. (2026). Distributive justice in the classroom: Teacher reasoning about student turn allocation decisions in whole-classroom discussions. *Learning, Culture and Social Interaction*, 59, 101031. <https://doi.org/10.1016/j.lcsi.2026.101031>

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R: Conceptualization, methodology, investigation, data curation, formal analysis, and writing of the original draft.
MM: Investigation, validation, research implementation, data collection, and review and editing of the manuscript.
Both authors reviewed and approved the final version of the manuscript.

AI DISCLOSURE STATEMENT

During the preparation of this manuscript, the authors used **ChatGPT by OpenAI** to assist with academic language editing, improvement of clarity and coherence, organization of the manuscript, and consistency of academic presentation. The tool was not used to generate or alter research data, statistical results, participant information, or empirical findings. All AI-assisted outputs were critically reviewed, verified, and revised by the authors. The authors take full responsibility for the accuracy, integrity, interpretation, and final content of the manuscript.

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