Jupiter: Jurnal Pendidikan Terapan
Homepage : https://journal.diginus.id/jupiter/index
Email: jurnalpendidikanterapan@gmail.com

p-ISSN : 2985-3214; e-ISSN : 2964-3171
Jupiter, Vol. 4, No. 3, July 2026
Page 743-754

Development and Evaluation of SmartFraction: An Integrated Problem-Based Fraction Learning Media for Enhancing Elementary Students' Numeracy Skills

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ARTICLE INFO

Article history:
Submitted: July 03, 2026
Final Revised: July 16, 2026
Accepted: July 24, 2026
Published: July 31, 2026

Keywords:
Fraction Learning; Learning Media; Numeracy Skills; Problem-Based Learning; SmartFraction.

ABSTRACT

Purpose: Despite the widespread use of Problem-Based Learning (PBL), fraction manipulatives, and multiple mathematical representations, few learning media integrate these components to improve students' numeracy skills. Therefore, this study aimed to develop and evaluate SmartFraction, an integrated PBL-based fraction learning media by examining its validity, practicality, and effectiveness. **Methods:** This study employed a Research and Development approach using the ADDIE model, followed by a quasi-experimental pretest-posttest control group design. Participants comprised 23 fourth-grade students (12 experimental and 11 control). Data were collected through expert validation, response questionnaires, classroom observations, and numeracy tests. Data were analyzed using descriptive statistics, the Shapiro–Wilk test, Levene's test, an independent-samples *t*-test, normalized gain (N-Gain), and Hedges' *g*. **Findings:** SmartFraction demonstrated high validity (88.33%) and high practicality based on user responses. Students in the experimental group achieved significantly higher numeracy performance than those in the control group ($p < 0.001$). The experimental group achieved a moderate N-Gain (0.6487), whereas the control group achieved a low N-Gain (0.2387). Hedges' *g* (4.94) indicated a very large effect, although this result should be interpreted cautiously because of the relatively small sample. Overall, SmartFraction proved to be a valid, practical, and effective learning medium for improving students' numeracy skills. **Research Implications:** SmartFraction offers an effective alternative by integrating contextual problem solving, multiple fraction representations, and collaborative learning to strengthen numeracy. **Originality:** SmartFraction uniquely integrates PBL, multiple fraction representations, and numeracy-oriented tasks into a single learning medium, addressing a gap in previous fraction learning research.

Doi: https://doi.org/10.61255/jupiter.v4i3.1544

INTRODUCTION

Numeracy is widely recognized as one of the essential competencies for lifelong learning, informed decision-making, and active participation in the twenty-first century. Beyond performing arithmetic operations, numeracy enables individuals to interpret quantitative information, analyze real-world situations, evaluate evidence, and apply mathematical reasoning to solve contextual problems. The development of these competencies should begin in elementary school, where students establish the conceptual foundations required for subsequent mathematical learning and everyday problem-solving. Consequently, mathematics instruction should emphasize not only procedural fluency but also conceptual understanding, reasoning, and the ability to connect mathematical ideas with authentic contexts (Sigus & Mädamürk, 2025). However, strengthening numeracy remains a major challenge in Indonesia. According to the Programme for International Student Assessment (PISA, 2023), Indonesian students achieved an average mathematics score of 366, substantially below the (OECD, 2023) average of 472, indicating persistent weaknesses in mathematical reasoning and problem-solving. Although PISA assesses 15-year-old students, its findings reflect cumulative learning experiences developed throughout compulsory education and underscore the importance of establishing strong numeracy foundations from the primary grades (OECD, 2023). Therefore, improving elementary mathematics instruction through meaningful, contextual, and conceptually oriented learning experiences is essential to strengthen students' mathematical reasoning and support long-term numeracy development (Septianing, 2024).

Among the mathematical topics introduced in elementary school, fractions remain one of the most challenging concepts for students because they require conceptual reasoning beyond whole-number arithmetic. Unlike whole numbers, fractions represent a relationship between the numerator, denominator, referent whole, and unit fraction, all of which must be interpreted as a single mathematical quantity rather than as independent numbers. Many students, however, continue to exhibit whole-number bias, leading them to compare fractions by considering only the numerator or denominator without understanding their proportional relationship. This misconception also affects students' ability to estimate fraction magnitude, identify equivalent fractions, and justify mathematical comparisons in meaningful contexts. Consequently, students often rely on memorized procedures while demonstrating limited conceptual understanding of why a fraction represents a particular quantity or how it can be applied to solve real-life problems (Bouck et al., 2023). To overcome these difficulties, instruction should promote the coordination of numerator and denominator through multiple mathematical representations, including area models, fraction strips, number lines, and visual diagrams. These representations help students connect concrete experiences with symbolic notation, strengthen conceptual understanding, and support flexible reasoning across different fraction situations. Research has shown that learning environments incorporating multiple representations significantly improve students' understanding of fraction equivalence, comparison, composition, and operations by enabling them to examine the same mathematical idea from different perspectives rather than relying on a single procedure (Bruce et al., 2023).

To address these challenges, various instructional approaches have been developed to strengthen students' conceptual understanding of fractions and numeracy skills. Among them, Problem-Based Learning (PBL) has gained considerable attention because it engages students in authentic mathematical problems that require reasoning, inquiry, collaboration, and reflection rather than rote application of formulas. Through PBL, students identify known and unknown information, explore alternative solution strategies, justify their reasoning, and evaluate the appropriateness of their solutions, thereby fostering higher-order mathematical thinking (Santos-Trigo, 2024). A recent systematic review further confirmed that effective PBL implementation in K–12 mathematics consistently incorporates contextual problem solving, collaborative inquiry, creativity, and reflective learning, all of which contribute to meaningful mathematical understanding (Ayari et al., 2025). Contextual learning is particularly important because successful numeracy depends not only on procedural competence but also on students' ability to connect mathematical concepts with real-life situations. Students possessing stronger mathematical understanding also tend to demonstrate richer contextual knowledge, indicating that meaningful contexts support successful mathematical problem-solving (Saccardo et al., 2024). In addition, intervention studies have reported positive effects of carefully designed instructional resources. Whole-class fraction interventions supported students' conceptual understanding and fraction operations (Björkhammer et al., 2024), while research-based instructional resources improved teaching quality by providing structured learning tasks, multiple representations, and opportunities for teacher reflection and instructional support (Schoen et al., 2024). Although these studies demonstrate promising outcomes, their instructional focus remains relatively independent.

Despite these advances, several important gaps remain. Previous studies have demonstrated the effectiveness of concrete and virtual manipulatives, game-based learning, and digital media in supporting students' mathematical achievement and engagement. For example, blended learning environments combining physical and virtual manipulatives have been shown to improve conceptual understanding by integrating hands-on experiences with dynamic visual representations (Siller & Ahmad, 2024). Likewise, game-based mathematical activities can increase students' motivation, participation, and problem-solving skills by making abstract mathematical concepts more tangible and meaningful (Debrenti, 2024). Within the Indonesian context, Problem-Based Learning supported by Smart Apps Creator has also been reported to significantly improve students' fraction problem-solving ability compared with conventional PBL instruction (Malasari & Andrijati, 2024). However, these studies generally focused on improving learning outcomes through a single instructional strategy or technology without integrating contextual Problem-Based Learning, multiple fraction representations, collaborative mathematical reasoning, and numeracy-oriented tasks into a coherent instructional medium. Furthermore, few studies have systematically evaluated such media through both product development procedures and controlled effectiveness testing. This gap is also reflected in the research setting. Preliminary classroom observations conducted at SDN 1 Sendangrejo revealed that 8 of 12 fourth-grade students (66.7%) scored below the minimum mastery criterion (KKM = 70) in fraction learning. Classroom observations further indicated that students experienced difficulties interpreting fractions in contextual problems, relied primarily on memorized procedures, and rarely justified their mathematical reasoning during classroom discussions. These findings highlight the need for an instructional medium that not only improves computational performance but also promotes conceptual understanding, mathematical reasoning, and contextual numeracy in elementary fraction learning.

Unlike previous studies that investigated Problem-Based Learning (PBL), manipulatives, multiple representations, or digital media as separate instructional approaches, SmartFraction integrates these components into a coherent learning medium designed specifically to strengthen elementary students' numeracy skills. The medium combines contextual PBL scenarios with multiple fraction representations, collaborative mathematical reasoning, guided inquiry, and reflective numeracy tasks within a structured learning sequence. Rather than functioning solely as a teaching aid, SmartFraction supports students in interpreting real-life situations, representing fractions using multiple models, comparing alternative solution strategies, justifying mathematical reasoning, and reflecting on the reasonableness of their solutions. In addition, the medium was systematically developed using the ADDIE model, ensuring that each instructional component was aligned with curriculum objectives, students' learning characteristics, and principles of effective mathematics instruction. Its quality was established through expert validation and user practicality evaluation, while its educational impact was examined through a quasi-experimental pretest-posttest control group design accompanied by normalized gain and Hedges' *g* effect-size analyses. Accordingly, this study contributes not only a validated instructional product but also empirical evidence demonstrating how the integration of contextual PBL, multiple representations, collaborative reasoning, and systematic instructional design can enhance elementary students' conceptual understanding of fractions and numeracy skills.

METHOD

Research Design

A Research and Development (R&D) approach combined with a quasi-experimental pretest-posttest control group design was employed to develop and evaluate SmartFraction, an integrated Problem-Based Learning (PBL)-based fraction learning medium for improving elementary students' numeracy skills. Product development followed the ADDIE model, comprising analysis, design, development, implementation, and evaluation. Selection of the ADDIE model was based on its systematic and iterative framework, which facilitates alignment between instructional needs, product design, classroom implementation, and continuous improvement through formative evaluation (Kökver et al., 2025). Effectiveness evaluation employed a quasi-experimental design involving an experimental class that learned using SmartFraction integrated with PBL and a control class that received conventional fraction instruction. Intact classrooms were retained to preserve authentic classroom conditions while enabling rigorous comparison of students' numeracy outcomes under equivalent learning objectives and instructional duration (Cortez et al., 2023).

Development and evaluation were conducted in two complementary phases. The development phase focused on designing, validating, revising, and implementing SmartFraction through the ADDIE framework, whereas the evaluation phase examined product quality and instructional effectiveness. Product quality was determined through expert validation, teacher and student response questionnaires, and classroom implementation observations, while instructional effectiveness was evaluated using pretest-posttest scores, normalized gain (N-Gain), and Hedges' *g* effect size. Integrating systematic instructional development with experimental evaluation provided comprehensive evidence of SmartFraction's validity, practicality, and effectiveness in enhancing fourth-grade students' numeracy skills. Table 1 summarizes the overall research design by outlining the major research phases, principal activities, and expected outcomes, thereby illustrating the relationship between product development and effectiveness evaluation.

Table 1. Overview of the Research Design

Research Phase	Main Activities	Expected Outcomes
Development Phase	Analysis, design, development, implementation, and evaluation using the ADDIE model	Valid and practical SmartFraction learning medium
Evaluation Phase	Quasi-experimental pretest-posttest control group design	Evidence of SmartFraction effectiveness in improving students' numeracy skills

Table 1 illustrates that the research comprised two complementary phases. The development phase emphasized producing a valid and practical learning medium through the ADDIE model, whereas the evaluation phase examined its instructional effectiveness using a quasi-experimental design. Together, these phases established a coherent methodological framework that enabled a comprehensive evaluation of SmartFraction's validity, practicality, and effectiveness. To provide a clearer overview of the research procedure, Figure 1 presents the sequential framework of the study, illustrating the integration of the ADDIE development process with the quasi-experimental evaluation. This framework demonstrates how each stage contributed systematically to the development, validation, implementation, and evaluation of SmartFraction.

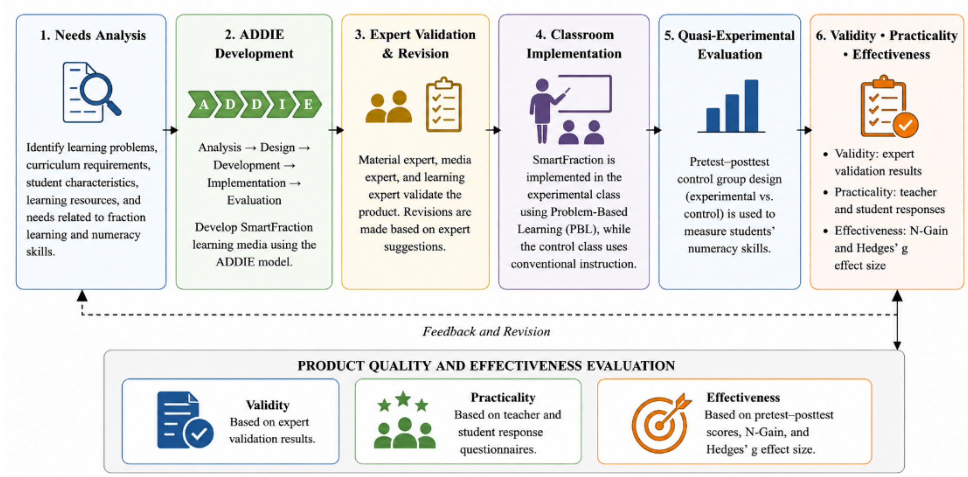

Figure 1. Overall Research Framework

Figure 1 highlights the logical progression of the research from needs analysis to effectiveness evaluation. Each phase generated outputs that informed the subsequent stage, ensuring methodological continuity throughout the study. This integrated framework strengthened the alignment between instructional development, product validation, classroom implementation, and effectiveness testing, thereby providing comprehensive evidence of SmartFraction's validity, practicality, and effectiveness.

Participants

Participants comprised 23 fourth-grade students from SDN 1 Sendangrejo, Blora Regency, Central Java, Indonesia, during the 2025/2026 academic year. One intact class consisting of 12 students was assigned as the experimental group, whereas another intact class of 11 students served as the control group. Random assignment of individual students was not feasible because the study was conducted under authentic classroom conditions. Therefore, intact classrooms were retained to preserve regular instructional activities while allowing comparison of learning outcomes between groups (Matolić et al., 2023). Both groups studied the same fraction topics, followed the same curriculum, received equivalent instructional time, and completed identical pretest and posttest assessments. Baseline equivalence between groups was established using pretest scores before the intervention. The characteristics of participants assigned to the experimental and control groups are summarized in Table 2.

Table 2. Characteristics of Research Participants

Characteristic	Experimental Group (n = 12)	Control Group (n = 11)
School	SDN 1 Sendangrejo	SDN 1 Sendangrejo
Grade Level	Fourth Grade	Fourth Grade
Academic Year	2025/2026	2025/2026
Learning Topic	Fractions	Fractions
Learning Approach	SmartFraction integrated with Problem-Based Learning	Conventional instruction
Research Design	Pretest–Posttest	Pretest–Posttest
Assessment	Identical pretest and posttest	Identical pretest and posttest

Table 2 shows that both groups were comparable in terms of school setting, grade level, academic year, learning topic, and instructional duration. The only difference between the groups was the instructional approach, with the experimental group receiving SmartFraction integrated with Problem-Based Learning and the control group receiving conventional instruction. This design ensured that differences in learning outcomes could be attributed primarily to the intervention rather than to variations in participant characteristics or learning conditions.

Product quality was established through expert validation involving specialists with complementary expertise in mathematics education, instructional media, and elementary pedagogy. Each expert evaluated different aspects of SmartFraction to ensure its content validity, instructional quality, technical feasibility, and curriculum alignment before classroom implementation. The profiles and evaluation responsibilities of the expert validators are presented in Table 3.

SmartFraction Development
Development of SmartFraction followed the ADDIE model, comprising five iterative phases: Analysis, Design, Development, Implementation, and Evaluation. Each phase generated specific outputs that informed the subsequent phase, ensuring systematic product development and comprehensive evaluation before classroom implementation (Kanaki & Kalogiannakis, 2023).

Analysis Phase
Classroom observations, teacher interviews, and curriculum document analysis were conducted to identify instructional needs, curriculum requirements, and students' difficulties in learning fractions. The analysis revealed persistent difficulties in interpreting fractions within contextual situations, excessive reliance on memorized procedures, and limited conceptual understanding. Findings from this phase produced instructional specifications, including learning objectives, essential fraction concepts, contextual problem scenarios, and pedagogical requirements that guided the development of SmartFraction. These specifications established the foundation for the design phase by ensuring alignment between curriculum expectations, learner characteristics, and instructional needs (Kökver et al., 2025).

Design Phase
Instructional specifications derived from the analysis phase were translated into a comprehensive instructional blueprint comprising a storyboard, interface prototype, navigation structure, learning sequence, assessment plan, and Problem-Based Learning (PBL) scenarios. Contextual problem situations, multiple fraction representations, collaborative inquiry, mathematical reasoning, and reflective numeracy tasks were systematically integrated to ensure coherence among instructional objectives, curriculum requirements, and students' cognitive characteristics. This phase produced a complete prototype specification, which served as the design reference for SmartFraction development and ensured consistency between identified learning needs and instructional solutions (Kökver et al., 2025).

Development Phase
Development activities produced the initial SmartFraction prototype by integrating learning materials, visual illustrations, interactive fraction representations, learning activities, and assessment components into a unified instructional medium. Validation involved one mathematics education expert, one instructional media expert, and one elementary learning expert, who evaluated content accuracy, instructional quality, media design, technical functionality, and curriculum alignment. Recommendations from the experts were incorporated through iterative revisions until the product satisfied the predetermined validity criteria. This phase produced a validated SmartFraction prototype, providing evidence that the learning medium met the required pedagogical and technical standards before classroom implementation.

Implementation Phase
Classroom implementation involved applying the validated SmartFraction prototype in the experimental class following the Problem-Based Learning sequence, whereas the control class received conventional fraction instruction with equivalent learning objectives and instructional time. Teacher response questionnaires, student response questionnaires, classroom observation records, and learning achievement data were collected to evaluate implementation fidelity, usability, classroom feasibility, and student engagement. This phase generated empirical evidence regarding the practicality and classroom feasibility of SmartFraction, demonstrating its suitability for integration into elementary mathematics instruction (Ayari et al., 2025).

Evaluation Phase
Evaluation integrated formative and summative procedures to determine the overall quality of SmartFraction. Formative evaluation verified product validity through expert review and iterative revision, whereas summative evaluation examined instructional effectiveness following classroom implementation. Statistical analyses were performed using descriptive statistics, prerequisite assumption tests, normalized gain (N-Gain), and Hedges' *g* effect size to evaluate improvements in students' numeracy skills. This phase generated empirical evidence of SmartFraction's validity, practicality, and effectiveness, confirming its quality as an instructional medium and its effectiveness in improving fourth-grade students' numeracy skills.

Table 4 summarizes the implementation of the ADDIE model by presenting the principal activities, outputs generated at each phase, and their contributions to the overall study. The summary demonstrates how each phase systematically informed the subsequent stage, ensuring coherence throughout the development and evaluation of SmartFraction (Yilmaz et al., 2024).

Multiple research instruments were employed to comprehensively evaluate the quality and effectiveness of SmartFraction in accordance with the study objectives. Instrument selection was aligned with the evaluation framework covering validity, practicality, and effectiveness, while all validation sheets and questionnaires underwent expert review before classroom implementation to ensure content relevance, clarity, and instructional appropriateness. The numeracy achievement test was developed based on the fourth-grade mathematics curriculum and administered as equivalent pretest and posttest instruments. Table 5 summarizes the research instruments, the constructs measured, respondents, and their contributions to the study.

Table 5. Research Instruments and Their Contributions to the Study

Instrument	Construct Measured	Respondents	Contribution to the Study
Expert validation sheet	Content validity, instructional quality, media design, curriculum alignment	One mathematics education expert, one instructional media expert, one elementary learning expert	Verified the validity of SmartFraction before classroom implementation
Teacher response questionnaire	Practicality, usability, instructional feasibility	Classroom teacher	Evaluated the practicality of SmartFraction during implementation
Student response questionnaire	Usability, engagement, learning experience	Experimental group students	Evaluated students' perceptions of SmartFraction and supported practicality evidence
Classroom observation sheet	Implementation fidelity, student participation, learning activities	Observer	Verified the consistency and feasibility of classroom implementation
Numeracy achievement test (pretest-posttest)	Numeracy skills and learning achievement	Experimental and control groups	Determined the effectiveness of SmartFraction in improving students' numeracy skills

As shown in Table 5, each instrument was designed to evaluate a distinct aspect of SmartFraction while collectively providing complementary evidence regarding its quality. Expert validation established product validity before implementation, teacher and student questionnaires verified practicality, classroom observations confirmed implementation fidelity, and numeracy achievement tests measured instructional effectiveness. The integration of these instruments ensured that the evaluation comprehensively addressed the study objectives and provided robust evidence supporting the validity, practicality, and effectiveness of SmartFraction.

Data Collection

Data collection was conducted sequentially throughout the development and evaluation of SmartFraction. During the development stage, expert validation was undertaken to evaluate content accuracy, instructional quality, media design, and curriculum alignment, followed by iterative revisions based on the experts' recommendations. During classroom implementation, teacher response questionnaires, student response questionnaires, and classroom observations were administered to examine the practicality and implementation fidelity of the learning medium. To evaluate instructional effectiveness, equivalent pretest and posttest numeracy assessments were administered to both the experimental and control groups before and after the intervention. Collecting data through multiple sources enabled triangulation of findings and provided comprehensive evidence regarding the validity, practicality, and effectiveness of SmartFraction.

Data Analysis

Data analysis was performed using descriptive and inferential statistical procedures according to the study objectives. Validation, practicality, and implementation data were first summarized descriptively to determine product quality, whereas students' numeracy achievement data were subsequently analyzed to examine instructional effectiveness. Prior to hypothesis testing, assumption tests were conducted using the Shapiro–Wilk test for normality and Levene's test for homogeneity to verify the suitability of parametric analysis. Differences in learning outcomes between the experimental and control groups were evaluated using an independent-samples *t*-test, while learning improvement was measured using the normalized gain (N-Gain). Hedges' *g* was calculated to estimate the magnitude of the intervention effect because it provides a less biased standardized effect size than Cohen's *d* for studies involving relatively small samples. Table 6 summarizes the analytical procedures applied to each research objective.

Table 6. Data Analysis Procedures

Research Objective	Data Source	Statistical Analysis	Expected Outcome
Evaluate product validity	Expert validation sheets	Percentage and validity criteria	Evidence of product validity
Evaluate product practicality	Teacher responses, student responses, classroom observations	Percentage and practicality criteria	Evidence of product practicality
Verify baseline equivalence	Pretest scores	Shapiro–Wilk, Levene's test, Independent-samples *t*-test	Comparable experimental and control groups
Measure learning improvement	Pretest and posttest scores	Normalized Gain (N-Gain)	Improvement in students' numeracy skills
Determine instructional effectiveness	Posttest scores	Independent-samples *t*-test and Hedges' *g*	Statistical significance and practical effect of SmartFraction

Table 6 demonstrates the methodological alignment between the research objectives, data sources, and statistical procedures. Applying analyses sequentially, from assumption testing to hypothesis testing and effect-size estimation provided robust evidence regarding the validity, practicality, and effectiveness of SmartFraction while ensuring the reliability and interpretability of the research findings.

Ethical Consideration

Approval to conduct the research was granted through an official Research Permit Letter issued by Universitas PGRI Semarang (UP.GRIS) (No. 2581/B.45/UPGRIS/IV/2026) before data collection commenced. Authorization to implement the research was subsequently obtained from the principal of SDN 1 Sendangrejo, Blora Regency, Central Java, Indonesia. Written informed consent was obtained from teachers, parents or legal guardians, and all participating students prior to participation. Voluntary participation, confidentiality, anonymity, and the right to withdraw at any stage without academic or personal consequences were ensured throughout the research process. All research data were anonymized, securely stored, and used solely for academic purposes in accordance with institutional regulations and internationally recognized ethical principles for educational research

RESULTS

Product Validity

Expert validation confirmed that SmartFraction achieved a high level of validity with respect to content quality, instructional design, media quality, and curriculum alignment before classroom implementation. Evaluation by one mathematics education expert, one instructional media expert, and one elementary learning expert consistently indicated that the developed learning medium met the predefined quality criteria for elementary fraction instruction. The overall validation results are presented in Table 2.

Table 2. Expert Validation Results

Validator	Score (%)	Validity Category
Mathematics education expert	88.00	Highly valid
Instructional media expert	87.00	Highly valid
Elementary learning expert	90.00	Highly valid
Average	88.33	Highly valid

Validation scores ranged from 87.00% to 90.00%, yielding an overall validity score of 88.33%, which was classified as highly valid. Feedback from the experts primarily emphasized refinement of visual presentation, instructional guidance, and contextual learning activities to further improve classroom usability. These recommendations were incorporated through iterative revisions before implementation. Collectively, the validation findings demonstrated that SmartFraction fulfilled the predefined validity criteria and was suitable for classroom implementation, providing robust evidence of product validity prior to effectiveness evaluation.

Practicality of SmartFraction

SmartFraction demonstrated a high level of classroom practicality, indicating that the learning medium was readily accepted by both teachers and students during fraction instruction. Overall practicality findings are presented in Table 3.

Table 3. Practicality Results of SmartFraction

Respondent	Mean Score (%)	Practicality Category
Teacher	92.00	Highly practical
Students	93.00	Highly practical
Average	92.50	Highly practical

Teacher and student evaluations consistently identified the clarity of instructional activities, contextual problem-solving tasks, collaborative learning processes, and visual fraction representations as the principal strengths of SmartFraction. These features supported meaningful classroom interaction, facilitated conceptual understanding, and encouraged active student participation throughout the learning process. Collectively, the practicality findings confirmed that SmartFraction satisfied the predefined practicality criteria and was appropriate for routine implementation in elementary mathematics classrooms.

Classroom Implementation of SmartFraction

SmartFraction was implemented with a high level of instructional fidelity, demonstrating consistent adherence to the planned learning design throughout classroom instruction. Classroom observation results confirmed that the implementation achieved the predetermined quality criteria, indicating that the learning activities were conducted as intended. The overall implementation results are summarized in Table 4.

Table 4. Implementation Fidelity of SmartFraction

Implementation Component	Mean Score (%)	Category
Classroom implementation	94.81	Very well implemented

The implementation results demonstrated that the core instructional components of SmartFraction were consistently delivered throughout the learning process. Contextual problem-solving activities, collaborative investigation, mathematical discussion, and reflective learning were implemented effectively, promoting active student participation and meaningful conceptual understanding. Collectively, these findings confirmed that SmartFraction achieved a high level of implementation fidelity, providing strong evidence that the intervention was delivered consistently and in accordance with the intended instructional design.

Descriptive Statistics

Students' numeracy achievement exhibited comparable baseline performance between the experimental and control groups, whereas greater learning gains emerged in the experimental group after the intervention. Comparisons of pretest scores, posttest scores, and normalized gain (N-Gain) provided an overview of changes in learning achievement before inferential statistical analyses were conducted. The descriptive results are summarized in Table 5.

Table 5. Descriptive Statistics of Students' Numeracy Achievement

Variable	Experimental Group (n = 12)	Control Group (n = 11)
Pretest Mean	59.42	59.09
Posttest Mean	85.58	68.82
N-Gain	0.6487 (Moderate)	0.2387 (Low)

Both groups demonstrated nearly identical pretest mean scores, indicating comparable initial numeracy achievement before the intervention. After the intervention, the experimental group achieved a substantially higher posttest mean and a greater normalized gain than the control group, reflecting more pronounced improvement in students' numeracy achievement. Collectively, the descriptive findings demonstrated a clear pattern of greater learning improvement in the experimental group, providing an empirical basis for the subsequent inferential analyses.

Assumption Test Results

The data fulfilled all prerequisite assumptions for parametric analysis before hypothesis testing. Results of the normality and homogeneity tests are presented in Tables 6 and 7.

Table 6. Shapiro–Wilk Normality Test Results

Variable	Experimental Group (p)	Control Group (p)	Decision
Pretest	0.976	0.882	Normal
Posttest	0.910	0.744	Normal

N-Gain	0.601	0.348	Normal

Table 6 indicates that the pretest, posttest, and N-Gain data in both groups followed a normal distribution, satisfying the normality assumption for subsequent parametric analysis.

Table 7. Levene's Homogeneity Test Results

Variable	**F**	***p***	**Decision**
Pretest	0.299	0.590	Homogeneous
Posttest	1.647	0.213	Homogeneous

Table 7 shows that the variances of the experimental and control groups were homogeneous for both pretest and posttest scores, indicating that the homogeneity assumption was satisfied. Collectively, these findings confirmed that all prerequisite assumptions for parametric analysis were fulfilled, providing a valid statistical basis for subsequent hypothesis testing to evaluate the effectiveness of SmartFraction.

Results consistently demonstrated that SmartFraction improved fourth-grade students' numeracy skills beyond those achieved through conventional instruction. Compared with conventional learning, SmartFraction produced greater learning gains and a substantially larger educational effect. Collectively, these findings provide robust empirical evidence supporting SmartFraction as an effective integrated Problem-Based Learning medium for improving elementary students' numeracy skills.

DISCUSSION

Improvement in elementary students' numeracy requires instructional experiences that connect contextual problem-solving with meaningful mathematical representations rather than emphasizing procedural computation alone. Beyond demonstrating instructional effectiveness, the present study contributes evidence that integrating contextual Problem-Based Learning (PBL) with multiple mathematical representations provides a coherent instructional model for developing elementary numeracy. Improvement in students' numeracy skills is likely attributable to opportunities to construct mathematical meaning through contextual investigation, collaborative reasoning, and progressive representation of fraction concepts. Such learning experiences encourage students to relate concrete situations to symbolic reasoning, thereby reducing dependence on whole-number strategies. Viegut et al., (2023) similarly emphasized that fraction understanding develops from meaningful experiences with part–whole relationships, while (Tursynkulova et al., 2023) and (Ayari et al., 2025) demonstrated that PBL strengthens mathematical reasoning by engaging students in contextual inquiry and collaborative problem-solving.

Development of conceptual understanding is also influenced by the coordinated use of multiple mathematical representations throughout the learning process. Improvement in numeracy skills is likely attributable to the integration of concrete manipulatives, visual fraction models, symbolic notation, and structured inquiry, enabling students to interpret, compare, and justify fraction relationships from multiple perspectives. Such integration reduced reliance on procedural memorization and encouraged conceptual reasoning about fraction magnitude and equivalence. Powell & R, (2023) identified measurement-oriented fraction activities as an effective approach to strengthening fraction magnitude understanding, whereas (Yao & Gan, 2024) argued that conceptual learning becomes more robust when learners coordinate multiple representations systematically. Whereas previous studies primarily examined representations or Problem-Based Learning separately, the present study demonstrates the instructional value of integrating both components within a systematically developed learning medium. Alignment with Bruner's enactive–iconic–symbolic progression further explains how conceptual understanding developed gradually through successive representational stages.

Meaningful classroom interaction emerged as another important mechanism underlying the effectiveness of the developed medium. Collaborative discussion, explanation of solution strategies, and interpretation of visual representations encouraged students to communicate mathematical ideas while refining their reasoning through peer interaction. Bach et al., (2024) highlighted that explicit connections among mathematical representations strengthen mathematical communication, whereas (Yeo & Webel, 2024) showed that representation-based activities improve students' coordination of units, quantities, and fraction relationships. Educationally, these findings suggest that elementary mathematics instruction should move beyond procedural practice by integrating contextual investigation, collaborative reasoning, and mathematical communication as essential components of numeracy learning.

Contribution of the present study extends beyond confirming the effectiveness of a learning medium. Integration of contextual Problem-Based Learning, multiple mathematical representations, collaborative inquiry, and systematic instructional development offers an instructional framework that addresses conceptual understanding and numeracy simultaneously. Findings reported by (Rodrigues et al., 2024) and (Zhang et al., 2025) support the educational value of representation-based mathematics instruction, while (Anazifa & Djukri, 2023) emphasized that inquiry-oriented learning environments promote higher-order thinking through active reasoning. Distinct from previous studies, SmartFraction combines these complementary instructional components within a single empirically validated learning medium evaluated through product development and quasi-experimental testing, thereby providing stronger evidence of instructional effectiveness in elementary mathematics. Because the intervention was implemented in a single fourth-grade classroom with a relatively small sample, interpretation of the findings should be limited to comparable instructional contexts. Future studies are recommended to examine SmartFraction across different schools, larger samples, and longer intervention periods to evaluate the consistency and sustainability of its impact on students' numeracy development.

CONCLUSION

SmartFraction was successfully developed as an integrated Problem-Based Learning medium that strengthens fourth-grade students' numeracy through contextual problem-solving, multiple mathematical representations, collaborative inquiry, and structured instructional activities. Development using the ADDIE model produced a learning medium that fulfilled the required criteria of validity, practicality, implementation fidelity, and instructional effectiveness. Improvement in students' numeracy is attributable to the coordinated integration of contextual learning experiences, concrete and visual fraction representations, and collaborative mathematical reasoning, providing empirical evidence that systematic instructional design can effectively support conceptual understanding of fractions in elementary mathematics.

Contribution of the present study extends beyond the development of a learning medium by proposing an integrated instructional framework that combines Problem-Based Learning, multiple mathematical representations, and systematic instructional development within a single empirically validated product. Educationally, the findings provide practical guidance for elementary teachers seeking to strengthen students' numeracy through contextual and representation-based instruction. Generalization of the findings should be limited to instructional contexts comparable to the participating fourth-grade classroom. Future studies are recommended to examine SmartFraction across larger samples, multiple schools, longer intervention periods, and digital or hybrid learning environments to evaluate the consistency and long-term sustainability of its impact on elementary students' numeracy development.

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ACKNOWLEDGEMENT

The author sincerely thanks the supervisors, expert validators, school principals, teachers, students, and all parties who supported the research, data collection, product development, and manuscript preparation. Special appreciation is also extended to family and colleagues for their encouragement and assistance throughout the study.

AUTHOR CONTRIBUTION STATEMENT

DAS: conceptualization, methodology, data collection, data analysis, and writing of the original draft. JS: supervision, validation, and manuscript review and editing. M: supervision, methodological guidance, and manuscript review and editing.

AI DISCLOSURE STATEMENT

The authors used ChatGPT (OpenAI) during the preparation of this work for language refinement, structural organization, and drafting assistance. All generated content was thoroughly reviewed, verified, and edited by the authors, who take full responsibility for the accuracy and integrity of the publication.

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