

Shifts in Eighth-Grade Students' Mental Model Levels in Understanding Quadrilateral Concepts Through Group Discussion

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ABSTRACT

Purpose: The purpose of this study is to observe how the *mental level of* the student model experiences a shift in understanding the concept of a quadrangle through the process of group discussion. **Methods:** This study uses a qualitative approach involving 150 grade VIII students who have previously studied quadrilateral material. A written test on the concept of a quadrangle was given to all participants to determine the initial *mental level* of their model before the group discussion was conducted. **Findings:** Preliminary results showed that 12 students were at the *initial level*, 65 students at the first transition, 52 students at the synthetic level, 18 students at the II transition, and 3 students at the formal level. After the group discussion process, there was a shift in *the mental level of the model* in several subjects, namely two students from the initial level shifted to the synthetic level; one student from transition I to synthetic; one student from transition I to transition II; and two students from the synthetic level to transition II. Meanwhile, two students at the transition level II and two students at the formal level did not experience a shift. **Research implications:** These findings suggest that group discussions have the potential to facilitate the mental restructuring of students' *models* in understanding geometry concepts, especially quadrangles. In addition, this research emphasizes the importance of social interaction in helping students move from intuitive understanding to more formal understanding. **Originality:** This research contributes by mapping in detail the shift in the mental level of the student model after the group discussion, as well as offering empirical evidence regarding the effectiveness of discussion as a pedagogical strategy in learning geometry concepts.



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INTRODUCTION

Conceptual understanding is the basis and an important stage in the series of mathematics learning. Mathematical conceptual understanding is a conceptual understanding that is the center of attention in the development of mathematics curriculum in each country (Amreta & Anisah, 2023; Andamon & Tan, 2018). Students' ability to learn mathematics is directly related to their understanding of mathematical concepts and principles. Concepts are the basis for higher-order thinking processes, it can be interpreted that students who understand concepts well will be better able to generalize and transfer their knowledge than students who only memorize definitions. Many studies on conceptual understanding, one of which is the conceptual understanding of quadrilaterals. Quadrilaterals are part of geometry learning.

Learning geometry should begin with the most basic geometric tasks such as drawing, playing with geometric shapes, and naming geometric shapes (Sari et al., 2021). Learning geometric concepts is a complex process and as a component of the geometry curriculum, defining and classifying quadrilaterals is considered a difficult subject by many learners (Baktemur et al., 2021; Clements & Battista, 1992; Demirel & Dizman, 2025; Erez & Yerushalmy, 2016; Manero & Arnal, 2021).

Several researchers have conducted research on conceptual understanding of quadrilaterals, including Ozkan & Bal (2017) who studied the analysis of students' errors in their understanding of quadrilaterals. Budiarto et al. (2017) studied the testing of abstract thinking of mathematical concepts by considering previously constructed abstractions on the concept of quadrilaterals. Ulger & Broutin (2017) studied the understanding of prospective teachers to express conceptions, definitions of quadrilaterals and their internal relationships in figural and formal concepts in the case of parallelograms. From these studies, it can be concluded that conceptual understanding of quadrilaterals is very important to study. In this study, it does not focus on improving students' conceptual

understanding because this study does not involve many students in the class but only a few students who will later qualify as subjects.

The mathematical thinking process focuses on conceptual understanding. Mathematical thinking can be summarized as a method for accessing the unknown to the known, consisting of making assumptions, gathering evidence, and generalizing about the case. To be able to actively participate in the mathematical thinking process, mathematical abilities are needed and developed, one of which is through the formation of mental models (Baltaci, 2016). The formation of a complete student mental model is one of the roles of the teacher as an educator of students during learning. Teachers' teaching strategies and the processing of teaching materials significantly impact the development of students' mental models.

According to Duffy (2012), the concept of mental models was first proposed, according to the mind builds a small-scale model of reality that is used to anticipate an event, reason and to base an explanation. Mental models according to several other experts (Turk, 2016; Zimet, 2017), according to Zimet (2017) mental models are cognitive structures that have process or product properties. Mental models also have several functions that allow them to store data and implement strategies to obtain results. According to (Turk, 2016), mental models are internal and cognitive representations, are psychological representations of real and imaginary situations. Mental models are formed by people who understand and conceptualize events in the world. In their thinking, mental models are internal representations of the actual situation in people's thinking to understand. Their role is to account for reasoning both when they try to understand discourse and when they try to explain and predict the behavior of the world. Thus an analysis of mental models can give us very valuable information, understanding students' feelings and learning processes.

In this study, the researcher aims to reveal students' mathematical thinking processes in understanding the concept of quadrilaterals and the process of shifting their mental model levels through group discussions. Previous research related to the construction of the concept of quadrilaterals has not examined the leveling of mental models. Previous research on mental models has also only focused on leveling, without examining the shifting process. However, if the shifting process has been carried out, teachers can identify the location of student errors. Conceptual understanding cannot be separated from a person's mathematical thinking process, namely through their mental models (Agustina et al., 2022).

One way to stimulate students' thinking processes is by giving them a math problem. Mistakes made by students in working on math require attention, as they significantly impact their understanding of mathematical concepts. To minimize the impact of errors on constructing subsequent concepts, it is important to track the sources and causes of errors (Subanji & Nusantara, 2016). These sources can be found in the formation of students' thought patterns, known as the concept construction process. Understanding previous concepts is crucial for problem solving and therefore must be firmly embedded in students' long-term memory. If the concept is not firmly embedded in students' minds, it will not remain in their memory for long (Ncube & Luneta, 2025).

One good method for observing the process of shifting mental models is through discussion. In general, discussion can be considered an activity involving written or oral expression of different perspectives in a given situation. Appropriate discussion will help students achieve a critical understanding of a topic, self-awareness and the capacity for self-criticism, appreciation of diversity, and informed action (Abdulbaki et al., 2018). There are several definitions of discussion according to several experts. In learning, the group discussion method is a method in which two or more students express, present, explain, and argue their knowledge, experiences, opinions, and feelings (Elisabethangreiny & Saragih, 2025). Discussion is a situation in which students and students chat and share information, ideas, and opinions to solve a problem (Cruickshank et al., 2012). Discussion is a learning strategy that involves students sharing ideas on a common topic (Eggen & Kauchak, 2012). Discussion is a strategy that utilizes interactions between teachers and students and between students as the primary means of achieving learning objectives (Freitas et al., 2025). In learning, group discussions are small group discussions with four to six members. Small group discussions were chosen because they have been proven to be more effective than large class discussions (Stephen & Stephen, 2005).

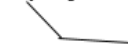
METHOD

This research uses a qualitative approach, this is in accordance with the characteristics of qualitative research described by Creswell (2012) as follows, namely the natural environment, the researcher as a key instrument, various data sources, inductive data analysis, meaning from participants, developing design, theoretical perspective, interpretive nature, and a holistic view.


This study uses a qualitative descriptive approach that examines existing problems and current work procedures. Qualitative descriptive research aims to describe what is currently happening. It involves describing, recording, analyzing, and interpreting current conditions. In other words, this qualitative descriptive study aims to obtain information about the existing situation. The purpose of this study is to describe or provide as accurate an overview as possible of students' mathematical thinking processes in understanding the concept of quadrilaterals from the perspective of mental models and how the process of shifting levels of mental models occurs through group discussions.


In this study, there are five components discussed in understanding the concept of quadrilaterals: the definition of quadrilaterals, types of quadrilaterals, shapes of quadrilaterals, properties of quadrilaterals, and quadrilateral story problems. The following questions were given to students:

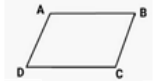
1.
 - a. Apa itu segiempat?
 - b. Lanjutkan gambar di bawah ini sehingga membentuk bangun segiempat!




(i)



(ii)
 2. Gambar dan sebutkan macam-macam segiempat!
 3. Gambar berikut adalah sebuah tangram, selidiki ada berapa bangun segiempat dalam tangram berikut!
- 
4.





 - a. Dari dua bangun di atas jelaskan apa persamaan dan perbedaannya?
 - b. Coba jelaskan sifat-sifat dari bangun di atas!
 5.
 - a. Gambariah bangun persegi panjang dengan luas yang sama tetapi kelilingnya berbeda!
 - b. Gambariah bangun persegi panjang yang luasnya lebih besar dari persegi panjang dengan ukuran $p = 9$ cm dan $l = 3$ cm, tetapi kelilingnya lebih kecil!
 - c. Gambariah bangun persegi panjang yang luasnya lebih kecil dari persegi panjang dengan ukuran $p = 8$ cm dan $l = 5$ cm, tetapi kelilingnya lebih besar!

Figure 1. Student questions

This study involved 150 eighth grade students who had received quadrilateral material in the previous class. A written test on the concept of quadrilateral was given to the 150 students. From the results, it was found that 12 students were at the initial level, 65 students were at the transition level I, 52 students were at the synthetic level, 18 students were at the transition level II and 3 students were at the formal level. The research subjects were taken as many as 10 people, namely 2 people each according to the five levels of mental models in understanding the concept of quadrilaterals. The selection of 2 people from each level of mental models in understanding the concept of quadrilaterals was because the analysis used in this study was a constant comparison analysis, where the researcher would analyze the level of mental models of each subject and the intersection obtained from the mental models of 2 subjects was the characteristic of the mental models of the subject group.

The research subjects were taken as many as 10 people, namely 2 people each according to the five levels of mental models in understanding the concept of quadrilaterals. The selection of 2 people from each level of mental models in understanding the concept of quadrilaterals was due to the analysis used in this study is a constant comparison analysis, where the researcher will analyze the level of mental models of each subject and the intersection obtained from the mental models of 2 subjects is the characteristic of the mental models of the subject group. After selecting 10 research subjects, the researcher conducted interviews related to the students' answers to the previous test. The interviews were conducted by interviewing the subjects one by one until the desired data was obtained. In this study, the researcher provided the subjects with as much space as possible to express their thoughts in free language. The location and time of data collection were also determined by the subjects themselves so that they felt comfortable and could express themselves freely. The steps taken to obtain research data included direct observation, where the researcher observed and observed the students' activities while completing the test questions. Second, the researcher conducted interviews with the research subjects. The data for this study were obtained from students' written answers obtained after they understood the concept of quadrilaterals and the results of interview transcripts.

Based on the interview results, the 10 subjects then engaged in a group discussion on the same topic: the concept of quadrilaterals. Following the discussion, the subjects were given a second test to determine whether their mental models had shifted. The test results were then analyzed, and follow-up interviews were conducted by the researchers to determine the subjects' shifts between the first and second tests.

RESULTS

When collecting data to determine the existence of mental model levels that are in accordance with the theory created and to determine how the mathematical thinking process is formed of these mental models. A written test on understanding the concept of quadrilaterals was given to 150 students. Of the 150 students, there were 12 students at the initial level, 65 students at the transition level I, 52 students at the synthetic level, 18 students at the transition level II and 3 students at the formal level. Next, two students were selected at each mental model level to be interviewed in depth about their thinking process and later given the same questions again to determine how

the process of shifting the mental model level after discussing with other students. Two students were chosen because for comparison at the same level later. The initials of the research subjects and each level of their mental models from each subject can be seen in Table 1 below:

Table 1. Subject Initials and Mental Model Level

| Nu. | Initials | Mental Model Level |
|-----|----------|------------------------------|
| 1. | FA | Initial Mental Model |
| 2. | ZR | Initial Mental Model |
| 3. | NA | Mental Model of Transition I |
| 4. | IF | Mental Model of Transition I |
| 5. | NF | Synthetic Mental Model |
| 6. | KH | Synthetic Mental Model |
| 7. | PY | Transition Mental Model II |
| 8. | RF | Transition Mental Model II |
| 9. | ZM | Formal Mental Model |
| 10. | VZ | Formal Mental Model |

During the data collection process, each subject was given two assignments: the first at the beginning of the mental model leveling process and the second after a group discussion with other subjects to determine the level shift process. Interviews were also conducted twice: the first to determine the thinking process at each level of the mental model and the second after the subjects had completed the group discussion. Data was considered unclear if students were still unsure about the given problem, namely whether its characteristics could be classified as fulfilling or not fulfilling one of the mental model indicators to be explored. If the data was unclear (not yet conclusive), data validation was performed using the triangulation method. If the data was clear (conclusive) but still desired further clarification, verification or further examination was performed. The following is a summary of the results of the written assignments, interviews, and analysis for 10 students who occupied each level of the mental model.

Table 2. Results for Subjects FA and ZR

| Questions with Quadrilateral Understanding Representation | | Interview Based On Questions With Quadrilateral Understanding Representation | | Conclusion |
|---|-----------------------|--|-----------------------|--|
| Component | Concept Understanding | Component | Concept Understanding | |
| Definition of a quadrilateral | Does not meet the | Definition of a quadrilateral | Does not meet the | The subject is at the Initial mental model level |
| Various types of quadrilaterals | Does not meet the | Various types of quadrilaterals | Does not meet the | |
| Rectangular shape | Does not meet the | Rectangular shape | Does not meet the | |
| Properties of quadrilaterals | Does not meet the | Properties of quadrilaterals | Does not meet the | |
| Quadrilateral comparison story problems | Does not meet the | Quadrilateral comparison story problems | Does not meet the | |

Table 3. Results for NA and IF Subjects

| Questions With Quadrilateral Understanding Representation | | Interview Based On Questions With Quadrilateral Understanding Representation | | Conclusion |
|---|-----------------------|--|-----------------------|--|
| Component | Concept Understanding | Component | Concept Understanding | |
| Definition of a quadrilateral | Fulfil | Definition of a quadrilateral | Fulfil | The subject is at the Transition I mental model level. |
| Various types of quadrilaterals | Does not meet the | Various types of quadrilaterals | Does not meet the | |
| Rectangular shape | Does not meet the | Rectangular shape | Does not meet the | |
| Properties of quadrilaterals | Does not meet the | Properties of quadrilaterals | Does not meet the | |
| Quadrilateral comparison story problems | Does not meet the | Quadrilateral comparison story problems | Does not meet the | |

Table 4. Results for Subjects NF and KH

| Questions with Quadrilateral Understanding Representation | | Interview based on questions with quadrilateral understanding representation | | Conclusion |
|---|-----------------------|--|-----------------------|---|
| Component | Concept Understanding | Component | Concept Understanding | |
| Definition of a quadrilateral | Fulfil | Definition of a quadrilateral | Fulfil | The subject is at the Synthetic mental model level. |
| Various types of quadrilaterals | Fulfil | Various types of quadrilaterals | Fulfil | |
| Rectangular shape | Does not meet the | Rectangular shape | Does not meet the | |
| Properties of quadrilaterals | Does not meet the | Properties of quadrilaterals | Does not meet the | |
| Quadrilateral comparison story problems | Does not meet the | Quadrilateral comparison story problems | Does not meet the | |

Table 5. Results for Subjects PY and RF

| Questions with Quadrilateral Understanding Representation | | Interview based on questions with quadrilateral understanding representation | | Conclusion |
|---|-----------------------|--|-----------------------|---|
| Component | Concept Understanding | Component | Concept Understanding | |
| Definition of a quadrilateral | Fulfil | Definition of a quadrilateral | Fulfil | The subject is at the Transition II mental model level. |
| Various types of quadrilaterals | Fulfil | Various types of quadrilaterals | Fulfil | |
| Rectangular shape | Fulfil | Rectangular shape | Fulfil | |
| Properties of quadrilaterals | Does not meet the | Properties of quadrilaterals | Does not meet the | |
| Quadrilateral comparison story problems | Does not meet the | Quadrilateral comparison story problems | Does not meet the | |

Table 6. Results for Subjects ZM and VZ

| Questions with Quadrilateral Understanding Representation | | Interview based on questions with quadrilateral understanding representation | | Conclusion |
|---|-----------------------|--|-----------------------|---|
| Component | Concept Understanding | Component | Concept Understanding | |
| Definition of a quadrilateral | Fulfil | Definition of a quadrilateral | Fulfil | The subject is at the Formal mental model level |
| Various types of quadrilaterals | Fulfil | Various types of quadrilaterals | Fulfil | |
| Rectangular shape | Fulfil | Rectangular shape | Fulfil | |
| Properties of quadrilaterals | Fulfil | Properties of quadrilaterals | Fulfil | |
| Quadrilateral comparison story problems | Fulfil | Quadrilateral comparison story problems | Fulfil | |

DISCUSSION

The leveling of mental models for students' thinking processes in understanding the concept of quadrilaterals has met the validity and reliability requirements according to the criteria set by researchers and has been validated by experts. In the description of the characteristics of students' thinking processes reviewed from the mental models above, there are main characteristics that differentiate each level of mental models. These differences lie in how students understand the concept of quadrilaterals, namely questions about the definition of quadrilaterals, types of quadrilaterals, quadrilateral shapes, quadrilateral properties and quadrilateral comparison story problems.

The research findings revealed a shift in mental model levels in understanding the concept of quadrilaterals after group discussions. These group discussions were conducted using a peer tutoring learning model. Two discussion groups were created, each containing subjects with each of the five levels of mental model. After the group discussions were completed, the 10 subjects were given the same problem again. The study then examined whether there was a shift in their mental model levels (either upward or downward) or whether the subjects remained at their original levels.

Based on the results of the research on students' thinking processes in understanding the concept of quadrilaterals, viewed from the initial mental model, students were unable to answer the definition of a quadrilateral perfectly, students only mentioned one element, namely its side, but could continue the picture of a quadrilateral. Students were unable to mention the various types of quadrilaterals as a whole, only able to answer shapes that were square or rectangular. Students were unable to answer how many quadrilaterals there were in a tangram, only mentioned the outer part of the tangram that was rectangular, students did not understand that the inside of the tangram could also be called a quadrilateral. Students were unable to answer about similarities and differences and were unable to explain the properties of parallelograms and trapezoids perfectly. Students were unable to provide examples of rectangles and their dimensions according to the question instructions. This initial mental model level is equivalent to the recognition level of the four levels of abstraction. At this recognition level, when students are given a problem or faced with a particular problem, they need the relationship rules that underlie the problem. To achieve this goal, they must recall the thinking structure they have acquired in previous activities to be able to use it in subsequent activities.

In the students' thinking process in understanding the concept of quadrilaterals, it is reviewed from the mental model of transition I. At this level, students are able to answer the definition of quadrilaterals well, namely by mentioning the existing elements including the sides and angles and continuing the quadrilateral image perfectly. Students are not able to mention the various types of quadrilaterals as a whole, can only answer the shapes that are square or rectangular. Students are not able to answer how many quadrilaterals there are in a tangram, which is only mentioned the outer shape of the tangram, which according to students is the only shape that exists. Students are able to answer about the similarities of two quadrilaterals but are not able to answer the differences and are not able to explain the properties of parallelograms and trapezoids perfectly. Students are not able to provide examples of rectangles and their dimensions according to the question instructions.

The next step is the students' thinking process in understanding the concept of quadrilaterals in terms of synthetic mental models. At this level, students are able to answer the definition of a quadrilateral by mentioning all its elements, namely sides and angles, and continue the picture of a quadrilateral perfectly. Students are able to name the various types of quadrilaterals as a whole and name the shapes. Students are able to answer how many quadrilaterals there are in a tangram but not perfectly, only mentioning three shapes for the first subject and four shapes for the second subject. Students are able to answer about the similarities and differences between the two quadrilateral shapes in the problem but are unable to explain the properties of parallelograms and trapezoids perfectly. Students are able to provide examples of rectangles and their dimensions according to the instructions of the problem but can only answer question (a) only, for (b) and (c) are not completed well. The synthetic mental model level is equivalent to the mental model leveling by [Kholis & Luayyin \(2025\)](#) at the medium level. Level the conceptual mental model into three categories: high, medium, and low mental models. At this medium level, students are able to synthesize a concept by referring to previous concepts, but students have not yet fully succeeded in synthesizing new concepts perfectly.

In the students' thinking process in understanding the concept of quadrilaterals viewed from the mental model of transition II, students are able to answer the definition of quadrilaterals by mentioning all of their elements, namely sides and angles, and continuing the quadrilateral drawing perfectly. Students are able to mention the various types of quadrilaterals as a whole and give names to each of them. Students are able to answer how many quadrilaterals there are in a tangram but not perfectly, only mentioning eight for the first subject and mentioning seven for the second subject. Students are able to answer about the similarities and differences between the two quadrilaterals and are able to explain the properties of parallelograms and trapezoids but not perfectly. Students are able to provide examples of rectangles and their dimensions according to the question instructions but only questions (a) and (c) for the first subject and questions (a) and (b) for the second subject.

The highest level of students' thinking process in understanding the concept of a quadrilateral in terms of mental models is formal. At this level, students are able to answer the definition of a quadrilateral well, namely by naming all its elements, sides and angles, and continuing the picture of a quadrilateral perfectly. Students are able to name the various types of quadrilaterals as a whole and also name each shape. Students are able to answer perfectly how many quadrilaterals there are in a tangram, namely ten. Students are able to answer about similarities and differences and are able to explain the properties of parallelograms and trapezoids perfectly. Students are able to provide examples of rectangles and their dimensions according to the instructions of the question. This formal mental model level is equivalent to the mental model level at the target mental model level. At this level, students are able to understand a concept perfectly and comprehensively.

After understanding the students' thinking process in understanding the concept of quadrilaterals from their mental models, students were then given directions to conduct a small-scale group discussion because the discussion members were only students who were at the level of that mental model. The theory used in this study is positioning theory ([Dejarnette, 2015](#)). Positioning theory offers a research lens that aims to understand the interpersonal relationships between students in a discussion group when they work together. The purpose of the discussion is to see how students interact in a group so that each student positions themselves through their interactions with other students.

During the discussion, students are positioned through interactions with other students. This type of positioning is referred to as interactive positioning (Davies & Harré, 1999). The exchange of information and actions that occur naturally during group discussions also serve to position students relative to one another. Interactive positioning is a two-way process as students position themselves and others. In this group discussion, students engage in collaborative learning about mathematical material. Collaborative learning implies that students develop a shared understanding of mathematics that will eventually lead to the joint production of ideas (Akosah, 2025). The understanding referred to in this study is the concept of a quadrilateral.

During the discussion, students were able to share information about the concept of quadrilaterals. Because the researcher did not assign marks to the answers given in the first test, the subjects did not know whether their work was correct or incorrect. However, the subjects with the highest levels began sharing material related to the concept of quadrilaterals, connecting with other students at each level of their mental model. After the discussion, students were given the same questions as in the first test to determine the results of their mental model level shifts. The results of the mental model level shifts are presented in Table 1.

Table 7. Results of Shifting Levels of Students' Mental Models

| Nu. | Subject | Mental Model Level Test 1 | Mental Model Level Test 2 |
|-----|---------|---------------------------|---------------------------|
| 1 | FA | Initials | Synthetic |
| 2 | ZR | Initials | Synthetic |
| 3 | NA | Transition I | Synthetic |
| 4 | IF | Transition I | Transition II |
| 5 | NF | Synthetic | Transition II |
| 6 | KH | Synthetic | Transition II |
| 7 | PY | Transition II | Transition II |
| 8 | RF | Transition II | Transition II |
| 9 | ZM | Formal | Formal |
| 10 | VZ | Formal | Formal |

Based on Table 7 above, it can be seen that two students, namely FA and ZR, who were at the initial level in the first test after group discussions and working on the questions again, were then at the synthetic level. Students began to be able to build knowledge by making connections between new information and concepts they already had (Omwirhiren, 2015). Changes occurred in the component of understanding quadrilaterals, which initially students only mentioned one element in their understanding, after the second test students were able to answer with two elements of quadrilaterals, namely sides and angles. In the component of various quadrilaterals, there was also a change, initially only drawing two shapes because in students' minds quadrilaterals were only pictures of squares and rectangles, after the second test they were able to draw all quadrilaterals. Furthermore, in the component of quadrilateral shapes, students who initially only answered there was only one shape in the tangram because in their minds quadrilaterals were only shapes outside the tangram, after the second test they understood that inside the tangram there were several quadrilaterals, so students could answer that there were shapes in the tangram even though they were not perfect. For the properties of quadrilaterals, students also experienced changes, from initially only stating the similarities between parallelograms and trapezoids to understanding the differences between the two after the second test, but not yet being able to fully state the properties of the two. Therefore, students FA and ZR, who initially had an initial mental model level, are now able to move to the synthetic mental model level.

In group discussions, the exchange of ideas or information is indeed very influential on the results of changes or shifts in thinking patterns in students (Tan et al., 2020). Next, two students NA and IF who were initially at transition level I after the second test there was a change to be at the synthetic mental model level for NA and transition II for IF. Changes occurred in the components of various quadrilateral shapes, which initially students only drew two quadrilateral shapes, namely squares and rectangles after the second test they drew all flat shapes. Changes also occurred in the components of quadrilateral shapes which initially only mentioned one shape in the tangram, now they can name several shapes although not perfectly. In the component of the properties of quadrilaterals, students also saw changes which initially only answered the similarities of parallelograms and trapeziums after the second test were able to answer the differences between the two shapes. For other properties, NA students could not mention their properties perfectly, but IF students could mention the properties of the two shapes from other elements although not perfectly. Additionally, student IF was able to correctly answer two story problems about quadrilaterals, which she initially could not. Therefore, both students NA and IF, who were at the transition I mental model level after the second test, are now at the synthetic mental model level for NA and transition II for IF.

Next, two students NF and KH who were initially at the synthetic level after group discussion and given their second test were at the transition II mental model level. Changes occurred in the components of the quadrilateral shapes, both students were able to name more quadrilateral shapes in the tangram. The next change was in the properties of the parallelogram and trapezium, both students were able to name other elements in the properties of

the two shapes, although not perfectly. And finally, the change was in the component of the quadrilateral story questions, which initially both students could not answer the story questions about quadrilaterals, after the second test they were able to answer two of the three questions correctly. So, two students NF and KH who were initially at the synthetic level after discussion and given their second test were at the same level, namely transition II.

Furthermore, two students PY and RF who were initially at the transitional mental model level II after the discussion and given the second test, both students remained at the transitional mental model level II. Just like the two students ZM and VZ who were initially at the highest mental model level after the discussion and given the second test, there was no change either, they remained at the highest mental model level, namely forma.

CONCLUSION

During the process of shifting their mental model levels, some students experienced changes or shifts in their mental model levels. This was because before the second test, a group discussion had been conducted with friends at the mental model level. Changes occurred in each component, seen from the shift from the initial mental model level to the second mental model. Two students at the initial model level experienced a shift in their mental model to the synthetic level. Two students at the transition mental model level I experienced a shift in their mental model, one student shifted to the synthetic mental model level and the other student shifted to the transition mental model level II. Two students at the synthetic mental model level shifted to the transition mental model level II. Meanwhile, two students at the transition model level II and two students at the formal mental model level did not experience a shift in their mental model level, all students remained at the initial mental model level.

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AUTHOR CONTRIBUTION STATEMENT

LA contributed to conceptualization, methodology, supervision, validation, writing original draft, and writing, review & editing. LA contributed to methodology, formal analysis, investigation, resources, and writing, review & editing. S contributed to data curation, investigation, project administration, and writing original draft. All authors have read and approved the final version of the manuscript.

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The authors used ChatGPT (OpenAI) during the preparation of this work for limited language editing and grammar improvement. After using the tool, the authors carefully reviewed and revised the manuscript and take full responsibility for the accuracy, integrity, and content of the publication.

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